17-09-20

Ch-6
Trignometric Ratios

★ Table

Degree →	0°	30°	45°	60°	90°
sin θ	$\sqrt{\frac{0}{4}} = 0$	$\sqrt{\frac{1}{4}} = \frac{1}{2}$	$\sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{4}}$ $\frac{\sqrt{3}}{2}$	$\sqrt{\frac{4}{4}} = 1$ ←
Cos θ	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0 ←
Tan θ	0	$1/\sqrt{3}$	1	$\sqrt{3}/1$	∞
cot θ	∞	$\sqrt{3}/1$	1	$1/\sqrt{3}$	0
Sec θ	1	$2/\sqrt{3}$	$\sqrt{2}/1$	$2/1$	∞ ←
Cosec θ	∞	$2/1$	$\sqrt{2}/1$	$2/\sqrt{3}$	1 ←

- $\tan\theta = \frac{\sin\theta}{\cos\theta}$ Ex. $\frac{0}{1} = 0$, $\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

- Sec θ = Inverse of Cos θ's value (same degree)

★ Exercise 6.1

Q: 1 Find the value of following:

1.) $2 \sin 45^\circ \cos 45^\circ$

Ans.) $2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{2}{2} = 1$ Ans.

2.) $\cos 45^\circ \cos 60^\circ - \sin 45^\circ \sin 60^\circ$

Ans.) $= \frac{1}{\sqrt{2}} \times \frac{1}{2} - \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$

$= \frac{1 - \sqrt{3}}{2\sqrt{2}}$ Ans.)

3.) $\sin^2 30^\circ + 2\cos^2 45^\circ + 3\tan^2 60^\circ$

$= \left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 3\left(\frac{\sqrt{3}}{1}\right)^2$

$= \frac{1}{4} + 2 \times \frac{1}{2} + 3 \times \frac{3}{1}$

$= \frac{1}{4} + \frac{1}{1} + \frac{9}{1}$

$= \frac{1 + 4 + 36}{4} = \frac{41}{4}$ Ans.)

4.) $3 \sin 60^\circ - 4 \sin^3 60^\circ$

$= 3 \times \frac{\sqrt{3}}{2} - 4\left(\frac{\sqrt{3}}{2}\right)^3$

$= \frac{3\sqrt{3}}{2} - \frac{4}{8} \times \frac{3\sqrt{3}}{2}$

$\frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0$

5) $\frac{5\cos^2 60^\circ + 4\sin^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 45^\circ}$

$\Rightarrow \frac{5\times\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$\Rightarrow \frac{5\times\frac{1}{4} + 4\times\frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$

$\Rightarrow \frac{\left[\frac{5}{4} + \frac{16}{3} - \frac{1}{1}\right]}{\frac{4}{4}} \Rightarrow \frac{15+64-12}{12} = \frac{79-12}{12} = \frac{67}{12}$

$\Rightarrow \frac{\frac{67}{12}}{\frac{1}{1}} \Rightarrow \frac{67}{12}\times\frac{1}{1}$

$\Rightarrow \frac{67}{12}$ Ans

6. $4\cot^2 45^\circ - \sec^2 60^\circ + \sin^2 60^\circ + \cos^2 90^\circ$

Ans) $= 4\times(1)^2 - (2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (0)^2$

$= \frac{4}{1} - \frac{4}{1} + \frac{3}{4} + \frac{0}{1}$

$= \frac{16-16+3+0}{4} = \frac{3}{4}$ Ans

7) $\frac{4}{\cot^2 30^\circ} + \frac{1}{\sin^2 30^\circ} - \cos^2 45^\circ$

$\Rightarrow \frac{4}{(\sqrt{3}/1)^2} + \frac{1}{(1/2)^2} - \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{4}{3} + 1 \times \frac{4}{1} - \frac{1}{2}$

$= \frac{4}{3} + \frac{4}{1} - \frac{1}{2}$

$= \frac{8 + 24 - 3}{6} = \frac{32 - 3}{6} = \frac{29}{6}$ Ans.

8) $\frac{\tan^2 60^\circ + 4\sin^2 45^\circ + \sin^2 90^\circ}{3\sec^2 30^\circ + \operatorname{cosec}^2 60^\circ - \cot^2 30^\circ}$

$\Rightarrow \frac{\left(\frac{\sqrt{3}}{1}\right)^2 + 4 \times \left(\frac{1}{\sqrt{2}}\right)^2 + (1)^2}{3 \times \left(\frac{2}{\sqrt{3}}\right)^2 + \left(\frac{2}{\sqrt{3}}\right)^2 - \left(\frac{\sqrt{3}}{1}\right)^2}$

$\Rightarrow \frac{3 + 2 \times \frac{1}{2} + 1}{3 \times \frac{4}{3} + \frac{4}{3} - 3}$

$\Rightarrow \frac{3 + 2 + 1}{\left[\frac{4}{1} + \frac{4}{3} - \frac{3}{1}\right]} \Rightarrow \frac{12 + 4 - 9}{3} = \frac{7}{3}$

$\Rightarrow \frac{6}{7/3} \Rightarrow 6 \times \frac{3}{7} \Rightarrow \frac{18}{7}$ Ans)

9) $\frac{\sin 30^\circ - \sin 90^\circ + 2\cos 0^\circ}{\tan 30^\circ \tan 60^\circ}$

Ans.) $\frac{\frac{1}{2} - 1 + 2 \times 1}{\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{1}}$

$\Rightarrow \frac{\frac{1}{2} - 1 + 2}{1} \Rightarrow \frac{1 - 2 + 4}{2}$

$\Rightarrow \frac{3}{2} \times 1 \Rightarrow \frac{3}{2}$ Ans.

10.) $\frac{2\tan 30^\circ}{1 - \tan^2 30^\circ}$

Ans.) $\Rightarrow \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} \Rightarrow \frac{\frac{2}{\sqrt{3}}}{\left[1 - \frac{1}{3}\right]} \rightarrow \frac{3-1}{3} = \frac{2}{3}$

$\Rightarrow \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} \Rightarrow \frac{\cancel{2}}{\sqrt{3}} \times \frac{3}{\cancel{2}}$

$\Rightarrow \frac{3}{\sqrt{3}} = \sqrt{3}$ Ans.

11) Find the value of x in the following:

(i) $\cos x = \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$

Ans.) $\cos x = \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}$

$\cos x = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$

$\cos x = \frac{\cancel{2}\sqrt{3}}{\cancel{4}_2}$; $\cos x = \frac{\sqrt{3}}{2}$

$\cos x = \cos 30^\circ$ Ans. $x = 30^\circ$ Ans

(ii) $\sin 2x = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

Ans. $= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{4} - \frac{1}{4}$

$= \frac{2}{4} = \frac{1}{2}$

$\sin 2x = \frac{1}{2}$; $\sin 30^\circ = \frac{1}{2}$

$2x = 30^\circ$

$x = \frac{30^\circ}{2}$

$x = 15^\circ$ Ans

(iii) $\sqrt{3} \tan 2x = \sin 30^\circ + \sin 45^\circ \cos 45^\circ + 2 \sin 90^\circ$

Ans $= \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + 2 \times 1$

$= \frac{1}{2} + \frac{1}{2} + \frac{2}{1}$

$= \frac{1+1+4}{2} = \frac{6}{2} = 3$

$\sqrt{3} \tan 2x = 3$

$\tan 2x = \frac{3}{\sqrt{3}} = \sqrt{3}$

$\tan 2x = \tan 60^\circ$

$2x = 60^\circ$

$x = \frac{60}{2} = 30^\circ$ Ans

Q: Prove that:

12) $\frac{\cos 30^\circ + \sin 60^\circ}{1 + \cos 60^\circ + \sin 30^\circ} = \frac{\sqrt{3}}{2}$

Ans) L.H.S. $\Rightarrow \frac{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}{1 + \frac{1}{2} + \frac{1}{2}} \Rightarrow \frac{\frac{2\sqrt{3}}{2}}{\frac{2+1+1}{2}}$

$\Rightarrow \frac{\sqrt{3}}{2}$

$\Rightarrow \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{2}$

L.H.S. $\frac{\sqrt{3}}{2}$ = R.H.S $\frac{\sqrt{3}}{2}$ (H.P.)

13.) $4\cot^2 45^\circ - \sec^2 60^\circ - \sin^2 30^\circ = -1/4$

Ans.) L.H.S. $4 \times (1)^2 - (2)^2 - (1/2)^2$

$4 - 4 - 1/4$

$= -1/4$

L.H.S $-1/4$ = R.H.S. $-1/4$ (H.P.)

14.) $4 \sin 30^\circ \sin^2 60^\circ + 3 \cos 60^\circ \tan 45^\circ - 2\sec^2 45^\circ - \operatorname{cosec}^2 90^\circ$

Ans.) $4 \times \frac{1}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2 + 3 \times \frac{1}{2} \times 1$ L.H.S

$= \frac{4}{2} \times \frac{3}{4} + \frac{3}{2}$

$= \frac{3}{2} + \frac{3}{2}$

$= \frac{6}{2} = 3$ L.H.S.

R.H.S. $= 2 \times \left(\frac{\sqrt{2}}{1}\right)^2 - (1)^2$

$= 2 \times \frac{2}{1} - 1 = \frac{4}{1} - 1$

$= \frac{4-1}{1} = 3$

L.H.S 1 = R.H.S 1 (H.P.)

15) $\text{cosec}^2 45^\circ \sec^2 30^\circ \sin^3 90^\circ \cos 60^\circ = 4/3$

Ans) L.H.S. $\left(\frac{\sqrt{2}}{1}\right)^2 \times \left(\frac{2}{\sqrt{3}}\right)^2 \times (1)^3 \times \frac{1}{2}$ =

$= \frac{2}{1} \times \frac{4}{3} \times 1 \times \frac{1}{2}$

$= \frac{4}{3}$

L.H.S 4/3 = R.H.S 4/3 (H.P.)

16.) $\frac{\sin 60^\circ + \sin 30^\circ}{\sin 60^\circ - \sin 30^\circ} = \frac{\tan 60^\circ + \tan 45^\circ}{\tan 60^\circ - \tan 45^\circ}$

Ans) L.H.S. $\Rightarrow \frac{\frac{\sqrt{3}}{2} + \frac{1}{2}}{\frac{\sqrt{3}}{2} - \frac{1}{2}} \Rightarrow \frac{\frac{\sqrt{3}+1}{2}}{\frac{\sqrt{3}-1}{2}}$

$\Rightarrow \frac{\sqrt{3}+1}{2} \times \frac{2}{\sqrt{3}-1} \Rightarrow \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$\left[\begin{array}{l} \sqrt{3} = \tan 60^\circ \\ 1 = \tan 45^\circ \end{array}\right]$

$\because, \frac{\tan 60^\circ + \tan 45^\circ}{\tan 60^\circ - \tan 45^\circ}$

L.H.S = R.H.S. (H.P.)

17) $2(\cos^2 45° + \tan^2 60°) - 6(\sin^2 45° - \tan^2 30°) = 6$

Ans) L.H.S. $2\left[\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}}{1}\right)^2\right] - 6\left[\left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2\right]$

$\Rightarrow 2\left[\frac{1}{2} + \frac{3}{1}\right] - 6\left[\frac{1}{2} - \frac{1}{3}\right]$

$\Rightarrow 2\left[\frac{1+6}{2}\right] - 6\left[\frac{3-2}{6}\right]$

$\Rightarrow 2\left[\frac{7}{2}\right] - 6\left[\frac{1}{6}\right]$

$\Rightarrow 7 - 1 = 6$

L.H.S 6 = R.H.S 6 Proved (H.P.)

18) $(\sec^2 30° + \operatorname{cosec}^2 45°)(2\cos 60° + \sin 90° + \tan 45°) = 10$

Ans.) L.H.S. $\left[\left(\frac{2}{\sqrt{3}}\right)^2 + \left(\frac{\sqrt{2}}{1}\right)^2\right]\left[2 \times \frac{1}{2} + 1 + 1\right]$

$\Rightarrow \left(\frac{4}{3} + \frac{2}{1}\right)\left(\frac{3}{1}\right)$

$\Rightarrow \left[\frac{4+6}{3}\right]\left[\frac{3}{1}\right] \Rightarrow \frac{10}{3} \times 3 \Rightarrow 10$

L.H.S 10 = R.H.S 10 (H.P.)

19) $(1 - \sin 45° + \sin 30°)(1 + \cos 45° + \cos 60°) = 7/4$

Ans) $\left(1 - \frac{1}{\sqrt{2}} + \frac{1}{2}\right)\left(1 + \frac{1}{\sqrt{2}} + \frac{1}{2}\right)$ ← L.H.S.

$\Rightarrow \left(\frac{3}{2} - \frac{1}{\sqrt{2}}\right)\left(\frac{3}{2} + \frac{1}{\sqrt{2}}\right)$

$\{(a-b)(a+b) = a^2 - b^2\}$

$\Rightarrow \left(\frac{3}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2 \Rightarrow \frac{9}{4} - \frac{1}{2}$

$\Rightarrow \frac{9-2}{4} = \frac{7}{4}$

L.H.S 7/4 = R.H.S 7/4 (H.P.)

20) $\cos^2 0° - 2\cot^2 30° + 3\operatorname{cosec}^2 90° = 2(\sec^2 45° - \tan^2 60°)$

Ans) $\Rightarrow (1)^2 - 2 \times (\sqrt{3})^2 + 3 \times (1)^2 \Leftarrow$ L.H.S.

$\Rightarrow 1 - 6 + 3 \Rightarrow -2.$

R.H.S. $\Rightarrow 2[(\sqrt{2})^2 - (\sqrt{3})^2]$

$\Rightarrow 2[2 - 3]$

$\Rightarrow 2(-1)$

$\Rightarrow -2$

L.H.S -2 = R.H.S -2 (H.P.)

21.) If $x = 30°$ then prove that:

(i) $\sin 3x = 3\sin x - 4\sin^3 x$

Ans.) L.H.S, $= \sin 3x$

$= \sin(3 \times 30')$

$= \sin 90' \Rightarrow 1$

R.H.S. $= 3\sin x - 4\sin^3 x$

$= 3\sin 30' - 4\sin^3 30'$

$= 3 \times \frac{1}{2} - 4 \times (\frac{1}{2})^3$

$= \frac{3}{2} - 4 \times \frac{1}{8}$

$= 3/2 - 1/2 \Rightarrow 2/2 = 1$

LHS (1) = R.H.S (1) [H.P.]

(ii) $\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

Ans) L.H.S. = $\tan(2\times 30^\circ)$

$= \tan 60^\circ \Rightarrow \sqrt{3}$

R.H.S. = $\dfrac{2\tan 30^\circ}{1-\tan^2 30^\circ} \Rightarrow \dfrac{2\times 1/\sqrt{3}}{1-(1/\sqrt{3})^2}$

$\Rightarrow \dfrac{2/\sqrt{3}}{1-1/3}$

$\Rightarrow \dfrac{2/\sqrt{3}}{2/3}$

$\Rightarrow \dfrac{2}{\sqrt{3}} \times \dfrac{3}{2}$

$\Rightarrow \dfrac{3}{\sqrt{3}} \Rightarrow \sqrt{3}$

L.H.S $\sqrt{3}$ = R.H.S $\sqrt{3}$ [H.P.]

(iii) $\sin x = \sqrt{\dfrac{1-\cos 2x}{2}}$

Ans) L.H.S = $\sin x \Rightarrow \sin 30^\circ \Rightarrow 1/2$

R.H.S. = $\sqrt{\dfrac{1-\cos 2\times 30^\circ}{2}} \Rightarrow \sqrt{\dfrac{1-\cos 60^\circ}{2}}$

$\Rightarrow \sqrt{\dfrac{1-1/2}{2}}$

$\Rightarrow \sqrt{\dfrac{1/2}{2}} \Rightarrow \sqrt{\dfrac{1\times 1}{2\times 2}}$

$\Rightarrow \dfrac{\sqrt{1}}{\sqrt{4}} = \dfrac{1}{2}$

L.H.S 1/2 = R.H.S 1/2 (H.P.)

(iv) $\cos 3x = 4\cos^3 x - 3\cos x$

Ans) L.H.S. = $\cos 3 \times 30^\circ \Rightarrow \cos 90^\circ$

$\Rightarrow 0$

R.H.S. = $4\cos^3 30^\circ - 3\cos 30^\circ$

$= 4 \times (\sqrt{3}/2)^3 - 3 \times \sqrt{3}/2$

$= 4 \times \frac{3\sqrt{3}}{8_2} - \frac{3\sqrt{3}}{2}$

$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} \Rightarrow 0$

LHS 0 = R.H.S 0 (H.P.)

22) If $A = 60^\circ$ and $B = 30^\circ$ then prove that:

$$\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

Ans) L.H.S. = $\cot(60^\circ - 30^\circ) \Rightarrow \cot 30^\circ \Rightarrow \sqrt{3}$

R.H.S. = $\frac{\cot 60^\circ \cot 30^\circ + 1}{\cot 30^\circ - \cot 60^\circ} \Rightarrow \frac{1/\sqrt{3} \times \sqrt{3} + 1}{\sqrt{3} - 1/\sqrt{3}}$

$\Rightarrow \frac{1+1}{\frac{3-1}{\sqrt{3}}} \Rightarrow \frac{2}{2/\sqrt{3}}$

$\Rightarrow \frac{2}{2} \times \sqrt{3}$

$\Rightarrow \sqrt{3}$

L.H.S $\sqrt{3}$ = R.H.S $\sqrt{3}$ Any. (H.P.).