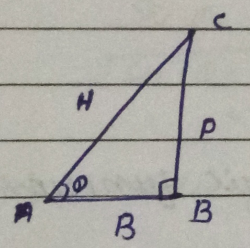
Ch-7

Trignometric Identities

★ Formulae:-

1. $\sin^2\theta + \cos^2\theta = 1$
2. $\sec^2\theta - \tan^2\theta = 1$
3. $\mathrm{cosec}^2\theta - \cot^2\theta = 1$

$$\underset{\mathrm{cosec}}{\overset{\sin}{\frac{P}{H}}} \qquad \underset{\sec}{\overset{\cos}{\frac{B}{H}}} \qquad \underset{\cot}{\overset{\tan}{\frac{P}{B}}}$$



Pythogorus theoram:-

$$(AC)^2 = (AB)^2 + (BC)^2$$

1.) • $\sin^2\theta + \cos^2\theta = 1$
• $\sin^2\theta = 1 - \cos^2\theta$
• $\cos^2\theta = 1 - \sin^2\theta$

2.) • $\sec^2\theta - \tan^2\theta = 1$
• $\sec^2\theta = 1 + \tan^2\theta$
• $\tan^2\theta = \sec^2\theta - 1$

3.) • $\mathrm{cosec}^2\theta - \cot^2\theta = 1$
• $\mathrm{cosec}^2\theta = 1 + \cot^2\theta$
• $\cot^2\theta = \mathrm{cosec}^2\theta - 1$

Value of Trignometric Ratios (T.R.) in form sin & cos θ

1. $\sin\theta = \dfrac{1}{\operatorname{cosec}\theta}$

2. $\cos\theta = \dfrac{1}{\sec\theta}$

3. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

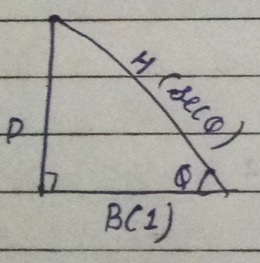
4. $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

5. $\operatorname{cosec}\theta = \dfrac{1}{\cos\theta}$

6. $\sec\theta = \dfrac{1}{\sin\theta}$

Q: 1.) Express all trignometric function in form of sec θ.

Ans



$\sec\theta = \dfrac{H}{B}$

$P \Rightarrow$ • $H^2 = P^2 + B^2$

$P^2 = H^2 - B^2$

$P^2 = (\sec\theta) - (1)^2$

$P^2 = \sec^2\theta - 1$

$P = \sqrt{\sec^2\theta - 1}$

$P = \sqrt{\sec^2\theta - 1}$, $B = 1$, $H = \sec\theta$

$\sin\theta = \frac{P}{H} = \frac{\sqrt{\sec^2\theta - 1}}{\sec\theta}$

$\cos\theta = \frac{B}{H} = \frac{1}{\sec\theta}$

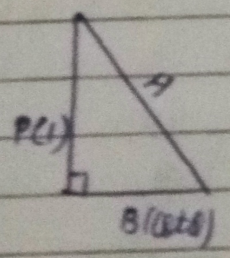
$\tan\theta = \frac{P}{B} = \frac{\sqrt{\sec^2\theta - 1}}{1}$

$\cot\theta = \frac{B}{P} = \frac{1}{\sqrt{\sec^2\theta - 1}}$

$\operatorname{cosec}\theta = \frac{H}{P} = \frac{\sec\theta}{\sqrt{\sec^2\theta - 1}}$

Exa Q:) Express all trignometric function in form of $\cot\theta$.

Ans



$\cot\theta = \frac{B}{P}$

$H \,?$ $\quad H^2 = P^2 + B^2$

$H^2 = (1)^2 + (\cot\theta)^2$

$H^2 = 1 + \cot^2\theta$

$H = \sqrt{1 + \cot^2\theta}$

$P = 1, \quad B = \cot\theta, \quad H = \sqrt{1 + \cot^2\theta}$

$\sin\theta = \frac{P}{H} = \frac{1}{\sqrt{1 + \cot^2\theta}}$

$\cos\theta = \frac{B}{H} = \frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$

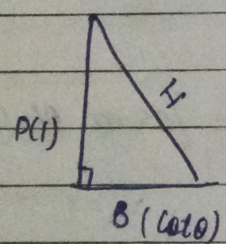
$\tan\theta = \frac{P}{B} = \frac{1}{\cot\theta}$

$\sec\theta = \frac{H}{B} = \frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$

$\operatorname{cosec}\theta = \frac{H}{P}$ $\frac{\sqrt{1+\cot^2\theta}}{1}$

Q22) Express T.R. $\sin\theta$, $\sec\theta$, $\tan\theta$ in term of $\cot\theta$.

Ans.



$\cot\theta = \frac{B}{P}$

$H \Rightarrow$ $H^2 = P^2 + B^2$

$H^2 = (1)^2 + (\cot\theta)^2$

$H^2 = 1 + \cot^2\theta$

$H = \sqrt{1+\cot^2\theta}$

$P = 1$, $B = \cot\theta$, $H = \sqrt{1+\cot^2\theta}$

$\sin\theta = \frac{P}{H} = \frac{1}{\sqrt{1+\cot^2\theta}}$

$\sec\theta = \frac{H}{B} = \frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$

Value of Trignometric Ratios (T.R.) in form sin & cos θ

1. $\sin\theta = \dfrac{1}{\operatorname{cosec}\theta}$

2. $\cos\theta = \dfrac{1}{\sec\theta}$

3. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

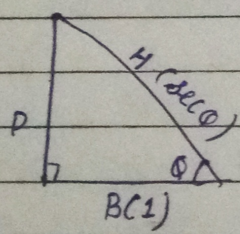
4. $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

5. $\operatorname{cosec}\theta = \dfrac{1}{\cancel{\sin\theta}\cos\theta}$

6. $\sec\theta = \dfrac{1}{\cancel{\cos\theta}\sin\theta}$

Q: 1.) Express all trignometric function in form of sec θ.

Ans



$\sec\theta = \dfrac{H}{B}$

$P \Rightarrow \cdot\ H^2 = P^2 + B^2$

$P^2 = H^2 - B^2$

$P^2 = (\sec\theta) - (1)^2$

$P^2 = \sec^2\theta - 1$

$P = \sqrt{\sec^2\theta - 1}$

$P = \sqrt{\sec^2\theta - 1}$, $B = 1$, $H = \sec\theta$

$$\sin\theta = \frac{P}{H} = \frac{\sqrt{\sec^2\theta - 1}}{\sec\theta}$$

$$\cos\theta = \frac{B}{H} = \frac{1}{\sec\theta}$$

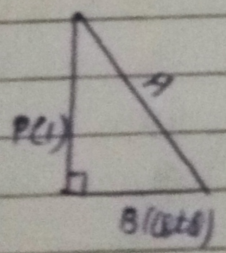
$$\tan\theta = \frac{P}{B} = \frac{\sqrt{\sec^2\theta - 1}}{1}$$

$$\cot\theta = \frac{B}{P} = \frac{1}{\sqrt{\sec^2\theta - 1}}$$

$$\csc\theta = \frac{H}{P} = \frac{\sec\theta}{\sqrt{\sec^2\theta - 1}}$$

Extra Q:) Expres all trignometric function in form of cot θ.

Ans



$$\cot\theta = \frac{B}{P}$$

H = $H^2 = P^2 + B^2$

$$H^2 = (1)^2 + (\cot\theta)^2$$

$$H^2 = 1 + \cot^2\theta$$

$$H = \sqrt{1 + \cot^2\theta}$$

$P = 1$, $B = \cot\theta$, $H = \sqrt{1+\cot^2\theta}$

$$\sin\theta = \frac{P}{H} = \frac{1}{\sqrt{1+\cot^2\theta}}$$

$$\cos\theta = \frac{B}{H} = \frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$$

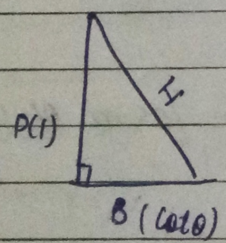
$$\tan\theta = \frac{P}{B} = \frac{1}{\cot\theta}$$

$$\sec\theta = \frac{H}{B} = \frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$$

$$\operatorname{cosec}\theta = \frac{H}{P} \quad \frac{\sqrt{1+\cot^2\theta}}{1}$$

Q22) Express T.R. $\sin\theta$, $\sec\theta$, $\tan\theta$ in term of $\cot\theta$.

Ans.



$$\cot\theta = \frac{B}{P}$$

$H \Rightarrow \quad H^2 = P^2 + B^2$

$H^2 = (1)^2 + (\cot\theta)^2$

$H^2 = 1 + \cot^2\theta$

$H = \sqrt{1+\cot^2\theta}$

$P = 1, \quad B = \cot\theta, \quad H = \sqrt{1+\cot^2\theta}$

$$\sin\theta = \frac{P}{H} = \frac{1}{\sqrt{1+\cot^2\theta}}$$

$$\sec\theta = \frac{H}{B} = \frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$$

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$\tan\theta = \frac{P}{B} = \frac{1}{\cot\theta}$

Q. Prove following with the help of identities

Q. 3.) $\cos^2\theta + \cos^2\theta \cot^2\theta = \cot^2\theta$

Ans.) L.H.S. $= \cos^2\theta(1+\cot^2\theta)$ $\{1+\cot^2\theta = \operatorname{cosec}^2\theta\}$

$= \cos^2\theta \times \operatorname{cosec}^2\theta$ $\{\operatorname{cosec}^2\theta = \frac{1}{\sin^2\theta}\}$

$= \cos^2\theta \times \frac{1}{\sin^2\theta}$

$= \frac{\cos^2\theta}{\sin^2\theta}$

$= \cot^2\theta$

L.H.S = R.H.S [H.P.]

Q. 4.) $\sec\theta(1-\sin\theta)(\sec\theta+\tan\theta) = 1$

Ans.) L.H.S $= \frac{1}{\cos\theta}(1-\sin\theta)\left(\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}\right)$

$= \cancel{\frac{1}{\cos}}\left(\frac{1-\sin\theta}{\cos\theta}\right)\left(\frac{1+\sin\theta}{\cos\theta}\right)$

$\{(a-b)(a+b) = a^2-b^2\}$

$= \frac{(1)^2-(\sin\theta)^2}{\cos^2\theta}$

$= \frac{1-\sin^2\theta}{\cos^2\theta}$ $\{1-\sin^2\theta = \cos^2\theta\}$

$= \frac{\cos^2\theta}{\cos^2\theta} = 1$

L.H.S = R.H.S [H.P.]

Q:5) $\operatorname{cosec}^2\theta + \sec^2\theta = \operatorname{cosec}^2\theta \cdot \sec^2\theta$

Ans.) L.H.S. $= \dfrac{1}{\sin^2\theta} + \dfrac{1}{\cos^2\theta}$

$= \dfrac{\cos^2\theta + \sin^2\theta}{\sin^2\theta\cos^2\theta}$ $\{\cos^2\theta + \sin^2\theta = 1\}$

$= \dfrac{1}{\sin^2\theta \cdot \cos^2\theta}$

$= \dfrac{1}{\sin^2\theta} \times \dfrac{1}{\cos^2\theta}$ $\left\{\dfrac{1}{\sin^2\theta} = \operatorname{cosec}^2\theta\right\}$

$\left\{\dfrac{1}{\cos^2\theta} = \sec^2\theta\right\}$

$= \operatorname{cosec}^2\theta \cdot \sec^2\theta$

L.H.S = R.H.S [H.P.].

Q:6.) $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$ Rationalise the denominator method

*

Ans.) L.H.S. $= \sqrt{\dfrac{1-\sin\theta}{1+\sin\theta} \times \dfrac{1-\sin\theta}{1-\sin\theta}}$

$\sqrt{\dfrac{(1-\sin\theta)^2}{(1)^2-(\sin\theta)^2}}$

$\sqrt{\dfrac{(1-\sin\theta)^2}{1-\sin^2\theta}}$ $\{1-\sin^2\theta = \cos^2\theta\}$

$\sqrt{\dfrac{(1-\sin\theta)^2}{\cos^2\theta}}$

$\dfrac{1-\sin\theta}{\cos\theta}$

$\dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}$ $\left\{\dfrac{1}{\cos\theta} = \sec\theta, \dfrac{\sin\theta}{\cos\theta} = \tan\theta\right\}$

$\sec\theta - \tan\theta$

L.H.S = R.H.S [H.P.]

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Q:7.) $\sqrt{\sec^2\theta + \csc^2\theta} = \tan\theta + \cot\theta$

Ans L.H.S. $= \sqrt{\dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta}}$

$\sqrt{\dfrac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \sin^2\theta}}$ $\{\sin^2\theta + \cos^2\theta = 1\}$

$\sqrt{\dfrac{1}{\cos^2\theta \sin^2\theta}}$

$\dfrac{1}{\cos\theta \sin\theta}$

$\dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta \cdot \sin\theta}$

$\dfrac{\sin^2\theta}{\cos\theta \sin\theta} + \dfrac{\cos^2\theta}{\cos\theta \sin\theta}$

$\dfrac{\sin\theta \times \cancel{\sin\theta}}{\cos\theta \times \cancel{\sin\theta}} + \dfrac{\cancel{\cos\theta} \times \cos\theta}{\cancel{\cos\theta} \times \sin\theta}$

$\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$ $\left\{\dfrac{\sin\theta}{\cos\theta} = \tan\theta ; \dfrac{\cos\theta}{\sin\theta} = \cot\theta\right\}$

$\tan\theta + \cot\theta$

L.H.S. = R.H.S. [H.P.]

Q:8.) $\dfrac{\tan\alpha + \tan\beta}{\cot\alpha + \cot\beta} = \tan\alpha \tan\beta$

Ans. $\left\{\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} ; \cot = \dfrac{\cos\alpha}{\sin\alpha}\right\}$

L.H.S $= \dfrac{\dfrac{\sin\alpha}{\cos\alpha} + \dfrac{\sin\beta}{\cos\beta}}{\dfrac{\cos\alpha}{\sin\alpha} + \dfrac{\cos\beta}{\sin\beta}} \Rightarrow \dfrac{\dfrac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\dfrac{\cos\alpha\sin\beta + \sin\alpha\cos\beta}{\sin\alpha\sin\beta}}$

$\Rightarrow \dfrac{\cancel{\sin\alpha\cos\beta + \cos\alpha\sin\beta}}{\cos\alpha\cos\beta} \times \dfrac{\sin\alpha\sin\beta}{\cancel{\cos\alpha\sin\beta + \sin\alpha\cos\beta}}$

$$= \frac{\sin\alpha \sin\beta}{\cos\alpha \cos\beta}$$

$$= \frac{\sin\alpha}{\cos\alpha} \times \frac{\sin\beta}{\cos\beta}$$

$$= \tan\alpha \cdot \tan\beta$$

L.H.S. = R.H.S. [H.P.].

Q:9) $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = 2\sec\theta$

Ans.) L.H.S $= \frac{(1+\sin\theta)^2 + (\cos\theta)^2}{\cos\theta(1+\sin\theta)}$

$$\left\{ \begin{aligned} (a+b)^2 &= a^2 + 2ab + b^2 \\ &= (1+\sin\theta)^2 = (1)^2 + 2 \times 1 \times \sin\theta + (\sin\theta)^2 \\ &= 1 + 2\sin\theta + \sin^2\theta \end{aligned} \right\}$$

$$= \frac{1 + 2\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$\{\sin^2\theta + \cos^2\theta = 1\}$

$$= \frac{1 + 2\sin\theta + 1}{\cos\theta(1+\sin\theta)}$$

$$= \frac{2 + 2\sin\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{2\cancel{(1+\sin\theta)}}{\cos\theta\cancel{(1+\sin\theta)}}$$

$$= \frac{2}{\cos\theta} = 2\sec\theta$$

L.H.S = R.H.S [H.P.].

Q:10) $\frac{\sin^4\theta - \cos^4\theta}{\sin^2\theta - \cos^2\theta} = 1$

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Ans.) L.H.S. = $\dfrac{(\sin^2\theta)^2 - (\cos^2\theta)^2}{\sin^2\theta - \cos^2\theta}$

$\{ (a^2 - b^2) = (a+b)(a-b) \}$

$= \dfrac{(\sin^2\theta + \cos^2\theta)\cancel{(\sin^2\theta - \cos^2\theta)}}{\cancel{\sin^2\theta - \cos^2\theta}}$

$= (\sin^2\theta + \cos^2\theta)$

$= 1$

L.H.S = R.H.S [H.P.]

Q: 11.) $\cot\theta - \tan\theta = \dfrac{1 - 2\sin^2\theta}{\sin\theta\cos\theta}$

Ans.) $\left\{ \cot\theta = \dfrac{\cos\theta}{\sin\theta} \;;\; \tan\theta = \dfrac{\sin\theta}{\cos\theta} \right\}$

L.H.S. $= \dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}$ $\{ \cos^2\theta = 1 - \sin^2\theta \}$

$= \dfrac{(1 - \sin^2\theta) - \sin^2\theta}{\sin\theta \cdot \cos\theta}$

$= \dfrac{1 - 2\sin^2\theta}{\sin\theta\cos\theta}$

L.H.S = R.H.S. [H.P.]

Q. 12.) $\cos^4\theta + \sin^4\theta = 1 - 2\cos^2\theta\sin^2\theta$.

Ans.) L.H.S. $= (\cos^2\theta)^2 + (\sin^2\theta)^2 + 2\cos^2\theta\sin^2\theta - 2\cos^2\theta\sin^2\theta$

$= (\cos^2\theta + \sin^2\theta)^2 - 2\cos^2\theta\sin^2\theta$

$= (1)^2 - 2\cos^2\theta\sin^2\theta$

$= 1 - 2\cos^2\theta\sin^2\theta$

L.H.S = RHS [H.P.]

Q:13) $(\sec\theta - \cos\theta)(\cot\theta + \tan\theta) = \tan\theta \cdot \sec\theta$.

Ans) $\left\{ \sec\theta = \frac{1}{\cos\theta} ; \cot\theta = \frac{\cos\theta}{\sin\theta} ; \tan\theta = \frac{\sin\theta}{\cos\theta} \right\}$

$$L.H.S. = \left(\frac{1}{\cos\theta} - \frac{\cos\theta}{1}\right)\left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right)$$

$$= \left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}\right)$$

$\{1 - \cos^2\theta = \sin^2\theta\}\{\cos^2\theta + \sin^2\theta = 1\}$.

$$= \frac{\sin^2\theta}{\cos\theta} \times \frac{1}{\sin\theta\cos\theta}$$

$$= \frac{\sin\theta \times \cancel{\sin\theta}}{\cos\theta} \times \frac{1}{\cancel{\sin\theta}\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta}$$

$\left\{ \frac{\sin\theta}{\cos\theta} = \tan\theta ; \frac{1}{\cos\theta} = \sec\theta \right\}$

$$= \tan \cdot \sec\theta$$

L.H.S. = R.H.S. [H.P.].

Q:14.) $\frac{1 - \tan^2\alpha}{\cot^2\alpha - 1} = \tan^2\alpha$

Ans.) $\left\{ \tan^2\alpha = \frac{\sin^2\alpha}{\cos^2\alpha} ; \cot^2\alpha = \frac{\cos^2\alpha}{\sin^2\alpha} \right\}$

$$L.H.S. \Rightarrow \frac{1 - \frac{\sin^2\alpha}{\cos^2\alpha}}{\frac{\cos^2\alpha}{\sin^2\alpha} - 1}$$

$$\Rightarrow \frac{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\alpha}}{\frac{\cos^2\alpha - \sin^2\alpha}{\sin^2\theta}}$$

$$= \frac{\cos^2\alpha - \sin^2\alpha}{\cos^2\alpha} \times \frac{\sin^2\alpha}{\cos^2\alpha - \sin^2\alpha}$$

$$= \frac{\sin^2\alpha}{\cos^2\alpha} \quad \left\{ \frac{\sin^2\alpha}{\cos^2\alpha} = \tan^2\alpha \right\}$$

$$= \tan^2\alpha$$

L.H.S. = R.H.S [H.P.].

Q:15) $\frac{\sin\theta}{1-\cos\theta} = \frac{1+\cos\theta}{\sin\theta}$

Ans.) ~~Sto~~ Rationalise the denominator method.

$$= \frac{\sin\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$$

$$= \frac{\sin\theta\,(1+\cos\theta)}{(1)^2 - (\cos\theta)^2}$$

$$= \frac{\sin\theta\,(1+\cos\theta)}{1-\cos^2\theta} \quad \{1-\cos^2\theta = \sin^2\theta\}$$

$$= \frac{\sin\theta\,(1+\cos\theta)}{\sin^2\theta}$$

$$= \frac{\cancel{\sin\theta}\,(1+\cos\theta)}{\cancel{\sin\theta} \times \sin\theta}$$

$$= \frac{1+\cos\theta}{\sin\theta}$$

L.H.S = R.H.S. [H.P].

16) $\sin^6\theta + \cos^6\theta = 1 - 3.\sin^2\theta\cos^2\theta$

Ans.) L.H.S. $= (\sin^2\theta)^3 + (\cos^2\theta)^3 \quad \{a^3+b^3 = (a+b)(a^2+b^2-ab\}$

$= (\sin^2\theta + \cos^2\theta)\left[(\sin^2\theta)^2 + (\cos^2\theta)^2 - \sin^2\theta\cos^2\theta\right]$

$\{\sin^2\theta + \cos^2\theta = 1\}$ $\{$ $a^2+b^2 = (a+b)^2\}$

$= (1)\left[(\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta\right]$

$= (1)\left[(\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta\right]$

$\{\sin^2\theta + \cos^2\theta = 1\}$

$= (1)[(1)^2 - 3\sin^2\theta\cos^2\theta]$

$= 1 - 3\sin^2\theta\cos^2\theta$

$\therefore$ L.H.S. = R.H.S. [H.P.]

17) $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \cancel{\tan\theta + \cot\theta}$ $\tan\theta + \cot\theta$

Ans) $\left\{\tan\theta = \dfrac{\sin\theta}{\cos\theta} ;\ \cot\theta = \dfrac{\sin\theta}{\cos\theta}\right\}$

$\Rightarrow \dfrac{\dfrac{\sin\theta}{\cos\theta}}{1-\dfrac{\cos\theta}{\sin\theta}} + \dfrac{\dfrac{\cos\theta}{\sin\theta}}{1-\dfrac{\sin\theta}{\cos\theta}}$

$= \dfrac{\dfrac{\sin\theta}{\cos\theta}}{\dfrac{\sin\theta-\cos\theta}{\sin\theta}} + \dfrac{\dfrac{\cos\theta}{\sin\theta}}{\dfrac{\cos\theta-\sin\theta}{\cos\theta}}$

$= \dfrac{\sin\theta}{\cos\theta} \times \dfrac{\sin\theta}{\sin\theta-\cos\theta} + \dfrac{\cos\theta}{\sin\theta} \times \dfrac{\cos\theta}{\cos\theta-\sin\theta}$

$= \dfrac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \dfrac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$

$= \dfrac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \dfrac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$

$= \dfrac{\sin^3\theta - \cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

$\{a^3 - b^3 = (a-b)(a^2+b^2+ab)\}$

$= \dfrac{\cancel{(\sin\theta - \cos\theta)}\,(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)}{\sin\theta\cos\theta\,\cancel{(\sin\theta - \cos\theta)}}$

$\dfrac{\cancel{\sin^2\theta}^{\sin\theta}}{\sin\theta\,\cos\theta} + \dfrac{\cancel{\cos^2\theta}^{\cos\theta}}{\sin\theta\cos\theta} + \dfrac{\cancel{\sin\theta\cos\theta}}{\cancel{\sin\theta\cos\theta}}$

$= 1 + \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} \left\{ \dfrac{\sin\theta}{\cos\theta} = \tan\theta ; \dfrac{\cos\theta}{\sin\theta} = \cot\theta \right\}$

$= 1 + \tan\theta + \cot\theta$

$\therefore$ L.H.S = R.H.S. [H.P].

18.) $\sin\theta(1+\tan\theta) + \cos\theta(1+\cot\theta) = \operatorname{cosec}\theta + \sec\theta$

Ans.) $\sin\theta\left(1 + \dfrac{\sin\theta}{\cos\theta}\right) + \cos\theta\left(1 + \dfrac{\cos\theta}{\sin\theta}\right) \left\{ \tan\theta = \dfrac{\sin\theta}{\cos\theta} ; \cot\theta = \dfrac{\cos\theta}{\sin\theta} \right\}$

$\sin\theta\left(\dfrac{\cos\theta + \sin\theta}{\cos\theta}\right) + \cos\theta\left(\dfrac{\sin\theta + \cos\theta}{\sin\theta}\right)$

$\sin\theta\left(\dfrac{\sin\theta + \cos\theta}{\cos\theta}\right) + \cos\theta\left(\dfrac{\sin\theta + \cos\theta}{\sin\theta}\right)$

Take common :- $\sin\theta + \cos\theta$

$\sin\theta + \cos\theta\left(\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}\right)$

$\sin\theta + \cos\theta\left(\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right) \{\sin^2\theta + \cos^2\theta = 1\}$

$\sin\theta + \cos\theta\left(\dfrac{1}{\sin\theta\ \cos\theta}\right)$

$\dfrac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}$

$$\frac{\sin\theta}{\sin\theta\cos\theta} + \frac{\cos\theta}{\sin\theta\cos\theta}$$

$$\frac{1}{\cos\theta} + \frac{1}{\sin\theta} \quad \left\{ \frac{1}{\cos\theta} = \sec\theta \; ; \; \frac{1}{\sin\theta} = \operatorname{cosec}\theta \right\}$$

$$= \sec\theta + \operatorname{cosec}\theta$$

$\therefore$, L.H.S = R.H.S [H.P.]

19.) $\sin^2\theta \cos\theta + \tan\theta \sin\theta + \cos^3\theta = \sec\theta$

Ans.) $\sin^2\theta \cos\theta + \cos^3\theta + \tan\theta \sin\theta \quad \left\{ \tan\theta = \frac{\sin\theta}{\cos\theta} \right\}$

$$= \cos\theta(\sin^2\theta + \cos^2\theta) + \frac{\sin\theta}{\cos\theta} \times \sin\theta \quad \{ \sin^2\theta + \cos^2\theta = 1 \}$$

$$= \cos\theta(1) + \frac{\sin^2\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\cos\theta} \quad \{ \cos^2\theta + \sin^2\theta = 1 \}$$

$$= \frac{1}{\cos\theta} \quad \left\{ \frac{1}{\cos\theta} = \sec\theta \right\}$$

$$= \sec\theta$$

$\therefore$, L.H.S. = R.H.S. [H.P.].

20.) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$

Ans.) $\left\{ \tan\theta = \frac{\sin\theta}{\cos\theta} \; ; \; \cot\theta = \frac{\cos\theta}{\sin\theta} \right\}$

$$\Rightarrow \frac{\frac{\sin\theta}{\cos\theta}}{1 - \frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1 - \frac{\sin\theta}{\cos\theta}}$$

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$\Rightarrow \dfrac{\dfrac{\sin\theta}{\cos\theta}}{\dfrac{\sin\theta - \cos\theta}{\sin\theta}} + \dfrac{\dfrac{\cos\theta}{\sin\theta}}{\dfrac{\cos\theta - \sin\theta}{\cos\theta}}$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} \times \dfrac{\sin\theta}{\sin\theta - \cos\theta} + \dfrac{\cos\theta}{\sin\theta} \times \dfrac{\cos\theta}{\cos\theta - \sin\theta}$

$\Rightarrow \dfrac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} + \dfrac{\cos^2\theta}{\sin\theta(\cos\theta - \sin\theta)}$

$\Rightarrow \dfrac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} - \dfrac{\cos^2\theta}{\sin\theta(\sin\theta - \cos\theta)}$

$\Rightarrow \dfrac{\sin^3\theta - \cos^3\theta}{\sin\theta\cos\theta(\sin\theta - \cos\theta)}$ $\{a^3 - b^3 = (a-b)(a^2+b^2+ab)\}$

$\Rightarrow \dfrac{\cancel{(\sin\theta - \cos\theta)}(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)}{\sin\theta\cos\theta\cancel{(\sin\theta - \cos\theta)}}$ $\{\sin^2\theta + \cos^2\theta = 1\}$

$\Rightarrow \dfrac{1 + \sin\theta\cos\theta}{\sin\theta\cos\theta}$

$\Rightarrow \dfrac{1}{\sin\theta\cos\theta} + \dfrac{\sin\theta\cancel{\cos\theta}}{\sin\theta\cancel{\cos\theta}}$

$\Rightarrow 1 + \dfrac{1}{\sin\theta} \times \dfrac{1}{\cos\theta}$ $\left\{\dfrac{1}{\sin\theta} = \operatorname{cosec}\theta;\ \dfrac{1}{\cos\theta} = \sec\theta\right\}$

$\Rightarrow 1 + \operatorname{cosec}\theta . \sec\theta$

$\therefore$, L.H.S = R.H.S. [H.P].

21.) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$.

Ans.) L.H.S. $\{(a+b)^2 = a^2 + b^2 + 2ab\}$

$= (\sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A) + (\cos^2 A + \sec^2 A + 2\cos A \sec A)$

$\sec A = \dfrac{1}{\cos A}$

$\operatorname{cosec} A = \dfrac{1}{\sin A}$

$= \left(\sin^2 A + \operatorname{cosec}^2 A + 2\cancel{\sin A} \times \dfrac{1}{\cancel{\sin A}}\right) + \left(\cos^2 A + \sec^2 A + 2\cancel{\cos A} \times \dfrac{1}{\cancel{\cos A}}\right)$

$= (\sin^2 A + \operatorname{cosec}^2 A + 2\} + \{\cos^2 A + \sec^2 A + 2)$

$= 2+2+(\sin^2 A + \operatorname{cosec}^2 A + \cos^2 A + \sec^2 A)$

$\sin^2 A + \cos^2 A = 1$

$2+2+(1 + \operatorname{cosec}^2 A + \sec^2 A)$

$\{\operatorname{cosec}^2 A = 1 + \cot^2 A \; ; \; \sec^2 A = 1 + \tan^2 A\}$

$2+2+1+(1+\cot^2 A + 1 + \tan^2 A)$

$2+2+1+1+1+(\cot^2 A + \tan^2 A)$

$7 + \tan^2 A + \cot^2 A$

$\therefore$ L.H.S. = R.H.S. [H.P.].

22.) $\sin^8\theta - \cos^8\theta = (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta\cos^2\theta)$

Ans} L.H.S. $(\sin^4\theta)^2 - (\cos^4\theta)^2$ $\{a^2-b^2 = (a+b)(a-b)\}$

$(\sin^4 - \cos^4\theta)(\sin^4\theta + \cos^4\theta)$

$[(\sin^2\theta)^2 - (\cos^2\theta)^2](\sin^4\theta + \cos^4\theta)$ $\{a^2-b^2 = (a+b)(a-b)\}$

$(\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta)$ $\{\sin^2\theta + \cos^2\theta = 1\}$

$(\sin^2\theta - \cos^2\theta)(1)(\sin^4\theta + \cos^4\theta)$

$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta)^2 + (\cos^2\theta)^2]$ $\{a^2+b^2 = (a+b)^2\}$

$(\sin^2\theta - \cos^2\theta)[(\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta] - 2\sin^2\theta\cos^2\theta$

$(\sin^2\theta - \cos^2\theta)[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta]$

$(\sin^2\theta - \cos^2\theta)[(1)^2 - 2\sin^2\theta\cos^2\theta]$

$(\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\cos^2\theta)$

$\therefore$ L.H.S. = R.H.S. [H.P.]

23.) $\sqrt{\dfrac{\sec\theta + 1}{\sec\theta - 1}} = \cot\theta + \operatorname{cosec}\theta$

Ans L.H.S — Rationalise the denominator method.

$= \sqrt{\dfrac{\sec\theta+1}{\sec\theta-1} \times \dfrac{\sec\theta+1}{\sec\theta+1}}$ $\{(a-b)(a+b) = a^2-b^2\}$

$= \sqrt{\dfrac{(\sec\theta+1)^2}{\sec^2\theta - 1}}$ $\{\sec^2\theta - 1 = \tan^2\theta\}$

$= 2+2+(\sin^2 A + \operatorname{cosec}^2 A + \cos^2 A + \sec^2 A)$

$\sin^2 A + \cos^2 A = 1$

$2+2+(1+\operatorname{cosec}^2 A + \sec^2 A)$

$\{\operatorname{cosec}^2 A = 1+\cot^2 A \;;\; \sec^2 A = 1+\tan^2 A\}$

$2+2+1+(1+\cot^2 A + 1+\tan^2 A)$

$2+2+1+1+1+(\cot^2 A + \tan^2 A)$

$7 + \tan^2 A + \cot^2 A$

$\therefore$ L.H.S. = R.H.S. [H.P.].

22) $\sin^8\theta - \cos^8\theta = (\sin^2\theta - \cos^2\theta)(1-2\sin^2\theta\cos^2\theta)$

Ans) L.H.S. $(\sin^4\theta)^2 - (\cos^4\theta)^2$ $\{a^2-b^2 = (a+b)(a-b)\}$

$(\sin^4 - \cos^4\theta)(\sin^4\theta + \cos^4\theta)$

$[(\sin^2\theta)^2 - (\cos^2\theta)^2](\sin^4\theta+\cos^4\theta)$ $\{a^2-b^2 = (a+b)(a-b)\}$

$(\sin^2\theta - \cos^2\theta)(\sin^2\theta+\cos^2\theta)(\sin^4\theta+\cos^4\theta)$ $\{\sin^2\theta+\cos^2\theta = 1\}$

$(\sin^2\theta - \cos^2\theta)(1)(\sin^4\theta+\cos^4\theta)$

$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta)^2 + (\cos^2\theta)^2]$ $\{a^2+b^2 = (a+b)^2\}$

$(\sin^2\theta - \cos^2\theta)[(\sin^2\theta)^2+(\cos^2\theta)^2+2\sin^2\theta\cos^2\theta] - 2\sin^2\theta\cos^2\theta$

$(\sin^2\theta - \cos^2\theta)[(\sin^2\theta+\cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta]$

$(\sin^2\theta - \cos^2\theta)[(1)^2 - 2\sin^2\theta\cos^2\theta]$

$(\sin^2\theta - \cos^2\theta)(1-2\sin^2\theta\cos^2\theta)$

$\therefore$ L.H.S. = R.H.S. [H.P.]

23) $\sqrt{\dfrac{\sec\theta+1}{\sec\theta-1}} = \cot\theta + \operatorname{cosec}\theta$

Ans L.H.S — Rationalise the denominator method.

$= \sqrt{\dfrac{\sec\theta+1}{\sec\theta-1} \times \dfrac{\sec\theta+1}{\sec\theta+1}}$ $\{(a-b)(a+b) = a^2-b^2\}$

$= \sqrt{\dfrac{(\sec\theta+1)^2}{\sec^2\theta - 1}}$ $\{\sec^2\theta - 1 = \tan^2\theta\}$

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$$= \frac{(\sec\theta+1)^2}{\tan^2\theta}$$

$$= \frac{\sec\theta+1}{\tan\theta}$$

$$= \frac{\sec\theta}{\tan\theta} + \frac{1}{\tan\theta} \quad \left\{ \sec\theta = \frac{1}{\cos\theta} ; \tan\theta = \frac{\sin\theta}{\cos\theta} \right\}$$

$$= \frac{1}{\cancel{\cos\theta}} \times \frac{\cancel{\cos\theta}}{\sin\theta} + \frac{1}{\tan\theta}$$

$$= \frac{1}{\sin\theta} + \frac{1}{\tan\theta} \quad \left\{ \frac{1}{\sin\theta} = \operatorname{cosec}\theta ; \frac{1}{\tan\theta} = \cot\theta \right\}$$

$$= \operatorname{cosec}\theta + \cot\theta$$

$\therefore$ L.H.S. = R.H.S. [H.P.]

24.) $$\frac{(1+\cot\theta+\tan\theta)(\sin\theta-\cos\theta)}{\sec^3\theta-\operatorname{cosec}^3\theta} = \sin^2\theta\cos^2\theta$$

Ans.) L.H.S. $\left\{ \cot\theta = \frac{\cos\theta}{\sin\theta} ; \tan\theta = \frac{\sin\theta}{\cos\theta} ; \sec\theta = \frac{1}{\cos\theta} ; \operatorname{cosec}\theta = \frac{1}{\sin\theta} \right\}$

$$= \frac{\left(1+\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}\right)(\sin\theta-\cos\theta)}{\frac{1}{\cos^3\theta}-\frac{1}{\sin^3\theta}}$$

$$= \frac{\left(\frac{\sin\theta\cos\theta+\overbrace{\cos^2\theta+\sin^2\theta}^{1}}{\sin\theta\cos\theta}\right)(\sin\theta-\cos\theta)}{\frac{\sin^3\theta-\cos^3\theta}{\cos^3\theta\sin^3\theta}}$$

$\{\cos^2\theta+\sin^2\theta = 1\}$

$\{a^3-b^3 = (a-b)(a^2+b^2+ab)\}$

$$= \frac{\left(\frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}\right)(\sin\theta-\cos\theta)}{\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\cos^3\theta\sin^3\theta}}$$

$$= \frac{1+\sin\theta\cos\theta}{\cancel{\sin\theta\cos\theta}} \times \cancel{\sin\theta-\cos\theta} \times \frac{\sin^3\theta\cos^3\theta}{\cancel{(\sin\theta-\cos\theta)}(\underbrace{\sin^2\theta+\cos^2\theta}_{1}+\sin\theta\cos\theta)}$$

$$= \frac{\cancel{1+\sin\theta\cos\theta}}{\cancel{1+\sin\theta\cos\theta}} \times \sin^2\theta\cos^2\theta$$

$$= \sin^2\theta\cos^2\theta$$

$\therefore$ L.H.S. = R.H.S [H.P.].

25) $$\underbrace{\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}}_{\text{L.H.S}} = \underbrace{\frac{2}{1-2\cos^2\theta}}_{\text{Middle (M)}} = \underbrace{\frac{2}{2\sin^2\theta-1}}_{\text{R.H.S.}}$$

Ans) L.H.S. $= \frac{(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2}{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}$ $\left\{\begin{array}{l}(a+b)^2 = a^2+b^2+2ab \\ (a-b)^2 = a^2+b^2-2ab\end{array}\right\}$

$\{(a-b)(a+b) = a^2-b^2\}$

$$= \frac{(\underbrace{\sin^2\theta+\cos^2\theta}_{1}+2\sin\theta\cos\theta)+(\underbrace{\sin^2\theta+\cos^2\theta}_{1}-2\sin\theta\cos\theta)}{\sin^2\theta-\cos^2\theta}$$

$$= \frac{1+\cancel{2\sin\theta\cos\theta}+1-\cancel{2\sin\theta\cos\theta}}{\sin^2\theta-\cos^2\theta}$$

$$= \frac{2}{\sin^2\theta-\cos^2\theta} \quad \text{— (1)}$$

$\therefore$ L.H.S. = M

As we know $\cos^2\theta = 1 - \sin^2\theta$, Now we put this value in eq (1) $\cancel{= \dfrac{2}{\sin^2\theta \cdot \cos^2\theta}} = \dfrac{2}{\sin^2\theta - \cos^2\theta}$

$$\cancel{= \frac{2}{\sin^2\theta - (1 - \sin^2\theta}} = \frac{2}{\sin^2\theta - (1 - \sin^2\theta)}$$

$$= \frac{2}{\sin^2\theta - 1 + \sin^2\theta}$$

$$= \frac{2}{2\sin^2\theta - 1}, \quad \because \quad M = R.H.S.$$

Therefore, L.H.S. = M = R.H.S [H.P.].

26.) $\dfrac{\cos A}{1 - \tan A} + \dfrac{\sin A}{1 - \cot A} = \sin A + \cos A$.

Ans.) L.H.S. $\left\{ \tan A = \dfrac{\sin A}{\cos A} \; ; \; \cot A = \dfrac{\cos A}{\sin A} \right\}$

$$= \frac{\cos A}{1 - \frac{\sin A}{\cos A}} + \frac{\sin A}{1 - \frac{\cos A}{\sin A}}$$

$$= \frac{\cos A}{\frac{\cos A - \sin A}{\cos A}} + \frac{\sin A}{\frac{\sin A - \cos A}{\sin A}}$$

$$= \cos A \times \frac{\cos A}{\cos A - \sin A} + \sin A \times \frac{\sin A}{\sin A - \cos A}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A} \quad \{a^2 - b^2 = (a-b)(a+b)\}$$

$$= \frac{\cancel{(\cos A - \sin A)}(\cos A + \sin A)}{\cancel{\cos A - \sin A}}$$

$$= \cos A + \sin A$$

$\therefore$ L.H.S. = R.H.S. [H.P.]

27.) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

Ans) L.H.S. $\left\{ \operatorname{cosec} A = \dfrac{1}{\sin A} \; ; \; \sec A = \dfrac{1}{\cos A} \right\}$

$$\left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$\left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right) \quad \left\{ \begin{array}{l} 1 - \sin^2 A = \cos^2 A \\ 1 - \cos^2 A = \sin^2 A \end{array} \right\}$$

$$\frac{\cos^{\cancel{2}} A}{\cancel{\sin A}} \times \frac{\sin^{\cancel{2}} A}{\cancel{\cos A}}$$

$$\cos A . \sin A.$$

R.H.S. $\left\{ \tan A = \dfrac{\sin A}{\cos A} \; ; \; \cot A = \dfrac{\cos A}{\sin A} \right\}$

$$= \frac{1}{\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}}$$

$$= \frac{1}{\dfrac{\sin^2 A + \cos^2 A}{\cos A \sin A}} \quad \{\sin^2 A + \cos^2 A = 1\}$$

$$\frac{1}{\frac{1}{\cos A \sin A}} \Rightarrow 1 \times \frac{\cos A \sin A}{1}$$

$$= \cos A . \sin A$$

$\therefore$, L.H.S. = R.H.S. [HP]

28.) $$\frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^3\theta}{\sin\theta - \cos\theta} = 1 + \sin\theta\cos\theta$$

Ans) L.H.S. $\left\{ \tan\theta = \frac{\sin\theta}{\cos\theta} \right\}$

$$\frac{\cos^2\theta}{1 - \frac{\sin\theta}{\cos\theta}} + \frac{\sin^3\theta}{\sin\theta - \cos\theta}$$

$$\frac{\cos^2\theta}{\frac{\cos\theta - \sin\theta}{\cos\theta}} + \frac{\sin^3\theta}{\sin\theta - \cos\theta}$$

$$\cos^2\theta \times \frac{\cos\theta}{\cos\theta - \sin\theta} + \frac{\sin^3\theta}{\sin\theta - \cos\theta}$$

$$\frac{\cos^3\theta}{\cos\theta - \sin\theta} + \frac{\sin^3\theta}{\sin\theta - \cos\theta}$$

$$\frac{\sin^3\theta}{\sin\theta - \cos\theta} - \frac{\cos^3\theta}{\sin\theta - \cos\theta}$$

$$\frac{\sin^3\theta - \cos^3\theta}{\sin\theta - \cos\theta} \quad \{a^3 - b^3 = (a-b)(a^2+b^2+ab)\}$$

$$= \frac{\cancel{(\sin\theta - \cos\theta)}(\overbrace{\sin^2\theta + \cos^2\theta}^{1} + \sin\theta\cos\theta)}{\cancel{\sin\theta - \cos\theta}}$$

$$= 1 + \sin\theta\cos\theta$$

$\therefore$, L.H.S. = R.H.S. [H.P.].

30.) If $\frac{\cos A}{\cos B} = m$ and $\frac{\cos A}{\sin B} = n$ then, prove that $(m^2+n^2)\cos^2 B = n^2$

Ans.) $(m^2+n^2)\cos^2 B = n^2$

$$\left(\frac{\cos^2 A}{\cos^2 B} + \frac{\cos^2 A}{\sin^2 B}\right)\cos^2 B = \frac{\cos^2 A}{\sin^2 B}$$

$$\text{L.H.S.} = \frac{\cos^2 A\sin^2 B + \cos^2 A\cos^2 B}{\cancel{\cos^2 B}\sin^2 B} \times \cancel{\cos^2 B}$$

$$= \frac{\cos^2 A\sin^2 B + \cos^2 A\cos^2 B}{\sin^2 B}$$ - Take common - $\cos^2 B$

$$= \frac{\cos^2 A(\sin^2 B + \cos^2 B)}{\sin^2 B}$$ $\{\sin^2 B + \cos^2 B = 1\}$

$$= \frac{\cos^2 A \times 1}{\sin^2 B} = \frac{\cos^2 A}{\sin^2 B}$$

$\therefore$, L.H.S = R.H.S [H.P.].

29.) If $\sec\theta + \tan\theta = P$ then, prove that $\frac{P^2-1}{P^2+1} = \sin\theta$

Ans.) $$\frac{(\sec\theta+\tan\theta)^2 - 1}{(\sec\theta+\tan\theta)^2+1} \Rightarrow \frac{\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta - 1}{\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta + 1}$$

$$\Rightarrow \frac{\sec^2\theta - 1 + \tan^2\theta + 2\sec\theta\tan\theta}{\tan^2\theta + 1 + \sec^2\theta + 2\sec\theta\tan\theta}$$

$$\left\{\begin{array}{l}\sec^2\theta - 1 = \tan^2\theta \\ \tan^2\theta + 1 = \sec^2\theta\end{array}\right\}$$

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$\Rightarrow \dfrac{\tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta}{\sec^2\theta + \sec^2\theta + 2\sec\theta\tan\theta}$

$\Rightarrow \dfrac{2\tan^2\theta + 2\sec\theta\tan\theta}{2\sec^2\theta + 2\sec\theta\tan\theta}$

(Take common $2\tan\theta$, upper side & $2\sec\theta$, lower side)

$\Rightarrow \dfrac{\cancel{2}\tan\theta\,\cancel{(\tan\theta + \sec\theta)}}{\cancel{2}\sec\theta\,\cancel{(\sec\theta + \tan\theta)}}$

$\Rightarrow \dfrac{\tan\theta}{\sec\theta}$

$\Rightarrow \dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$

$\Rightarrow \dfrac{\sin\theta}{\cancel{\cos\theta}} \times \dfrac{\cancel{\cos\theta}}{1} \Rightarrow \sin\theta$

$\therefore$ L.H.S. = R.H.S. [H.P.].