Chapter – 12
Exponents and Powers

- Numbers with exponents obey the following laws of exponents.

  (a) $a^m \times a^n = a^{m+n}$

  (b) $a^m \div a^n = a^{m-n}$

  (c) $\left(a^m\right)^n = a^{mn}$

  (d) $a^m \times b^m = (ab)^m$

  (e) $a^0 = 1$

  (f) $\dfrac{a^m}{b^m} = \left(\dfrac{a}{b}\right)^m$

- Very small numbers can be expressed in standard form using negative exponents.
- Use of Exponents to Express Small Number in Standard form:

  (i) Very large and very small numbers can be expressed in standard form.

  (ii) Standard form is also called scientific notation form.

  (iii) A number written as $m \times 10^n$ is said to be in standard form if m is a decimal number such that $1 \le m < 10$ and n is either a positive or a negative integer.

- Examples: $150,000,000,000 = 1.5 \times 10^{11}$.
- Exponential notation is a powerful way to express repeated multiplication of the same number. For any non-zero rational number 'a' and a natural number n, the product a x a x a x .... x a(n times) = a^n.

  It is known as the nth power of 'a' and is read as 'a' raised to the power n'. The rational number a is called the base and n is called exponent.

Cer 1.2 :/

4. Evaluate:-

3^{-2}

$\left[a^{-n} = \dfrac{1}{a^n}\right]$

$3^{-2} = \dfrac{1}{3^2} \Rightarrow \dfrac{1}{9}$ Ans.

$(-4)^{-2}$

$(-4) \times (-4)$

$\left[a^{-n} = \dfrac{1}{a^n}\right]$

$\Rightarrow \dfrac{1}{-4^2} \Rightarrow \dfrac{1}{16}$

$\left(\dfrac{1}{2}\right)^{-5}$

$\left[a^{-n} = \dfrac{1}{a^n}\right]$

$\cdot \left(\dfrac{2}{1}\right)^5 \Rightarrow \dfrac{32}{1} = 32.$

Q. Simplify and express the result in power notation with positive exponent:-

(i) $(-4)^5 \div (-4)^8$

$(a)^m \div (a)^n = a^{m-n}$
$= -4^{5-8}$
$\Rightarrow -4^{(-3)}$

$\left[a^{-n} = \dfrac{1}{a^n} \right]$

$\dfrac{1}{(-4)^3}$

(ii) $\left(\dfrac{1}{2^3} \right)^2$

$\Rightarrow \dfrac{1^2}{(2^3)^2}$

$\Rightarrow \left(\dfrac{a}{b} \right)^m \Rightarrow \dfrac{a^m}{a^n}$

$\dfrac{1}{2^{3\times 2}} = \dfrac{1}{2^6}$

$\left[(a^m)^n = a^{m\times n} \right]$

(iii) $(-3)^4 \times \left(\dfrac{5}{3} \right)^4$

$\int \left(\dfrac{a}{b} \right)^m = \dfrac{a^m}{a^n}$

$(-3)^4 \times \dfrac{5^4}{3^4} \Rightarrow \left[(ab)^m = a^m b^m \right.$

$\left. 3^{(4-4)} \times 5^4 \right.$

$$\frac{3^0 \times 5^4}{1 \times 5^4} \qquad \{ a^0 = 1 \}$$

$$\Rightarrow 5^4$$

(IV) $\quad \left(3^{-7} \div 3^{-10} \right) \times 3^{-5}$

$$\left(a^m \div a^n = a^{m-n} \right)$$

$$\Rightarrow \quad 3^{-7-(-10)}$$

$$\Rightarrow \quad 3^{-7+10}$$

$$\Rightarrow \quad 3^3 \times 3^{-5}$$

$$\Rightarrow \quad 3^{(3+(-5))}$$

$$\Rightarrow \quad 3^{(3-5)}$$

$$\Rightarrow \quad 3^{-2}$$

$$\left(a^{-m} = \frac{1}{a^m} \right)$$

$$\left(3^{-2} = \frac{1}{3^2} \right)$$

(V) $\quad \dfrac{2^{-3} \times (-7)^{-3}}{(2 \times -7)^{-3}}$

$$\left[a^m \times b^m = (ab)^m \right]$$

$$\left[a^{-m} = \frac{1}{a^m} \right)$$

$$\Rightarrow \frac{1}{(-14)^3}$$

$$(2 \times -7)^{-3}$$
$$(-14)^{-3} \Rightarrow \frac{1}{(-14)^3}$$

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Q3. Find the value of :-

(i) $(3^0 + 4^{-1}) \times 2^2$

$\left(1 + \dfrac{1}{4^1}\right) \times 2^2$ $\left[a^{-n} = \dfrac{1}{a^n}\right]$

$\left(\dfrac{4+1}{4}\right) \times 2^2$

$\dfrac{5}{4} \times 2^2$

$\dfrac{5}{4} \times 4 = \boxed{5}$

Or

$\dfrac{5}{2^2} \times 2^2$ $\left[a^m \div a^n = a^{m-n}\right]$

$5 \times 2^{2-2}$

5×2^0 $[a^0 = 1]$

$5 \times 1 = \boxed{5}$

(ii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$

Sol. $\left[a^{-n} = \dfrac{1}{a^n}\right]$ $\left(\dfrac{1}{2^1} \times \dfrac{1}{4^1}\right) \div 2^{-2}$

$\dfrac{1}{8} \div \dfrac{1}{2^2}$ $\left[a^{-n} = \dfrac{1}{a^n}\right]$

$\dfrac{1}{8} \div \dfrac{1}{4}$

$\dfrac{1}{8} \times \dfrac{4}{1} = \dfrac{4}{8} = \boxed{\dfrac{1}{2}}$

$$\Rightarrow \left(\frac{1}{2^1} \times \frac{1}{4^1}\right) \div 2^{-2}$$

$$\left(\frac{1}{2} \times \frac{1}{2^2}\right) \div 2^{-2}$$

$$\frac{1}{2^3} \div 2^{-2}$$

$$2^{-3} \div 2^{-2} \quad \left(a^m \div a^n = a^{m-n}\right)$$

$$2^{-3-(-2)}$$

$$2^{-3+2}$$

$$2^{-1} \qquad \left[a^{-n} = \frac{1}{a^n}\right]$$

$$\boxed{\frac{1}{2}}$$

(iii) $$\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$$

Sol. $$\left[a^{-n} = \frac{1}{a^n}\right]$$

$$\left(\frac{2}{1}\right)^2 + \left(\frac{3}{1}\right)^2 + \left(\frac{4}{1}\right)^2$$

$$4 + 9 + 16 \implies 29.$$

(iv) $(3^{-1} + 4^{-1} + 5^{-1})^0$

Sol $\left(a^{-n} = \dfrac{1}{a^n}\right)$

$\left(\dfrac{1}{3^1} + \dfrac{1}{4^1} + \dfrac{1}{5^1}\right)^0$

$\left(\dfrac{1}{3} + \dfrac{1}{4} + \dfrac{1}{5}\right)^0$

$\left(\dfrac{20 + 15 + 12}{60}\right)^0$

$\left(\dfrac{47}{60}\right)^0 = 1 \quad [a^0 = 1] \quad$ As.

(v) $\left\{\left(\dfrac{-2}{3}\right)^{-2}\right\}^2$

Sol $\left(a^{-n} = \dfrac{1}{a^n}\right)$

$\left\{\left(\dfrac{-3}{2}\right)^2\right\}^2$

$\left(\dfrac{-3}{2}\right)^4 \quad = \quad \dfrac{81}{16}$ Ans

Q4. Evaluate:-

(i) $\dfrac{8^{-1} \times 5^3}{2^{-4}}$

Sol. $\left(a^{-n} = \dfrac{1}{a^n}\right)$

$\left(\dfrac{1}{8^1} \times 5^3\right) \div 2^{-4}$

$\left(\dfrac{1}{8^1} \times 5^3\right) \div \dfrac{1}{2^4}$ $2 \times 2 \times 2 \times 2 = 16$

$\left(\dfrac{1}{8} \times 125\right) \div \dfrac{1}{16}$

$\left(\dfrac{125}{8}\right) \div \left(\dfrac{1}{16}\right)$

$\dfrac{125}{8} \times \dfrac{16}{1} \quad \rightarrow 125 \times 2 = 250.$

(ii) $\left(5^{-1} \times 2^{-1}\right) \times 6^{-1}$

$\left(a^{-n} = \dfrac{1}{a^n}\right)$

$\left(\dfrac{1}{5^1} \times \dfrac{1}{2^1}\right) \times \dfrac{1}{6^1}$

$\left(\dfrac{1}{5} \times \dfrac{1}{2}\right) \times \dfrac{1}{6}$

$$\left(\frac{1}{5} \times \frac{1}{2}\right) \times \frac{1}{6}$$

$$\frac{1}{10} \times \frac{1}{6} = \frac{1}{60}$$

Q5. Find the value of m for which

$$5^m \div 5^{-3} = 5^5$$

Sol $(a^m \div a^n = a^{m-n})$

$$5^{m-(-3)} = 5^5$$

$$5^{m+3} = 5^5$$

Comparing exponents both sides, we get.

$$m+3 = 5$$
$$m = 5-3 = 2$$

Q6. Evaluate (i) $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$

Sol $\left(a^{-n} = \frac{1}{a^n}\right)$

$$\left(\left(\frac{3}{1}\right)^1 - \left(\frac{4}{1}\right)^1\right)^{+1}$$

$$(3 - 4)^1$$

$$-1 \quad \underline{Ans}$$

(ii) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$

Sol $\left(a^{-n} \times \frac{1}{a^n}\right)$

Sol. $\left(\dfrac{8}{5}\right)^7 \times \left(\dfrac{5}{8}\right)^4$

$\left(\dfrac{8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8}{5 \times 8 \times 8 \times 8 \times 5 \times 5 \times 5}\right) \times \left(\dfrac{8 \times 5 \times 5 \times 5}{8 \times 8 \times 8 \times 8}\right)$

$\dfrac{8 \times 8 \times 8}{5 \times 5 \times 5} = \dfrac{512}{125}$

Q7. Simplify :-

(i) $\dfrac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}}$ $\quad (t \neq 0)$

Sol. $\dfrac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}}$ $\qquad \left\{ \begin{array}{l} 25 = 5^2 \\ 10 \times 5 \times 2 \end{array} \right\}$

$\dfrac{5^2 \times t^{-4}}{5^{-3} \times 5 \times 2 \times t^{-8}}$

$\dfrac{5^2 \times t^{-4}}{5^{-3} \times 5 \times 2 \times t^{-8}}$

$\dfrac{5^{2-(-3)-1} \times t^{-4-(-8)}}{2} \qquad \left(a^m \div a^n = a^{m-n}\right)$

$\dfrac{5^{2+3-1} \times t^{-4+8}}{2}$

$$\frac{5^{2+3-1} \times t^{-4+8}}{2}$$

$$\frac{5^{5-1} \times t^4}{2}$$

$$\frac{5^4 \times t^4}{2} = \frac{625\, t^4}{2}$$

(ii)

$$\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}} \qquad \{125 = 5 \times 5 \times 5 = 5^3\}$$
$$\{10 = 2 \times 5\}$$

(d)
$$\frac{3^{-5} \times 2^{-5} \times 5^{-5} \times 5^3}{5^{-7} \times 2^{-5} \times 3^{-5}} \qquad \{6 = 2 \times 3\}$$

$$\Rightarrow \left[(ab)^m = a^m b^m \right]$$

$$\frac{3^{-5} \times 2^{-5} \times 5^{-5+3}}{5^{-7} \times 2^{-5} \times 3^{-5}}$$

$$\frac{3^{-5} \times 2^{-5} \times 5^{-2}}{5^{-7} \times 2^{-5} \times 3^{-5}} \qquad \left[a^m \div a^n = a^{m-n} \right]$$

$$\Rightarrow \frac{3^{-5-(-5)} \times 2^{-5-(-5)} \times 5^{-2-(-7)}}{3^{-5+5} \times 2^{-5+5} \times 5^{-2+7}}$$

$3^0 \times 2^0 \times 5^5$ $(a^0 = 1)$

$1 \times 1 \times 5^5$
$1 \times 1 \times 3125 = 3125.$

Ex - 12.2

Q1 Express the following numbers in standard form:.

(i) 0.000000 0000085
Sol $\frac{8.5}{10^{12}}$ $\left[a^{-m} = \frac{1}{a^m} \right]$ 8.5×10^{-12}

(ii) 0.000000000000942
$\frac{9.42}{10^{12}}$ $\left[a^{-m} = \frac{1}{a^m} \right]$ 9.42×10^{-12}

(iii) 60200000000000000
6.02×10^{15}

(iv) 0.00000000837
$\frac{8.37}{10^9}$ $\left[a^{-m} = \frac{1}{a^m} \right]$ 8.37×10^{-9}

(v) 31860000000
3.186×10^{10}

Q2. Express the following numbers in usual form:-

(i) 3.02×10^{-6}

Sol $\left[a^{-m} = \dfrac{1}{a^m}\right]$ $3.02 \times \dfrac{1}{10^6} = \dfrac{302}{100000000}$ 0.00000302

(ii) 4.5×10^4

Sol $4.5 \times 10000 \Rightarrow 45000$

(iii) 3×10^{-8}

Sol $\left[a^{-m} \times \dfrac{1}{a^m}\right] = 3 \times \dfrac{1}{10^8} = \dfrac{3}{100000000} \Rightarrow 0.00000003$

(iv) 1.001×10^9

Sol $1.001 \times 1000000000 \Rightarrow 1001000000$

(v) 5.8×10^{12}

Sol $5.8 \times 1000000000000 \Rightarrow 5800000000000$

(vi) 3.61492×10^6

Sol $3.61492 \times 1000000 \Rightarrow 3614920$

Q3. Express the number appearing in the following statements in Standard form:-

(i) 1 micron is equal to $\dfrac{1}{1000000}$ m.

Sol $\left(\dfrac{1}{10^6} = 1 \times 10^{-6} m.\right)$

(ii) Charge of an electron is 0.000,000,000,000,000,000l Coulombs.

Sol $\dfrac{1.6}{10^{19}}$ $\Rightarrow$ 1.6×10^{-19} $\left[a^{-m} = \dfrac{1}{a^m}\right]$

(iii) Size of a bacteria is 0.0000005 m.

Sol $\dfrac{5}{10^7}$ $\Rightarrow$ 5×10^{-7} $\left(a^{-m} = \dfrac{1}{a^m}\right)$

(iv) Size of a plant cell is 0.0000275 m.

Sol $\dfrac{1.275}{10^5}$ $= 1.275 \times 10^{-5}$ $\left(a^{-m} = \dfrac{1}{a^m}\right)$

(v) Thickness of if a thick paper is 0.07 mm.

Sol $\dfrac{7}{100}$ $\Rightarrow$ $\dfrac{7}{10^2}$ $\Rightarrow$ 7×10^{-2} $\left(a^{-m} = \dfrac{1}{a^m}\right)$

Q4. In a stack _____ Stack?

Sol. Thickness of 1 book = 20mm
Thickness of 5 book = 20 × 5 = 100mm
Thickness of 1 one paper sheet = 0.016mm
Thickness of 5 papers =) 0.016 × 5 =) 0.08mm
Total Thickness of a stack =) 100 + 0.08
=) 100.08mm

100.08mm =) 100
1.0008×10^{2} mm.