

11/12/20

Ch = 10

Circles

Ex = 10.1

Q: 1: Fill in the blanks-

- (i) The centre of a circle lies in the interior of the circle.
- (ii) A point, whose distance from the centre of a circle is greater than its radius lies in exterior of the circle.
- (iii) The longest chord of a circle is a diameter of the circle.
- (iv) An arc is semicircle when its ends are the ends of a diameter.
- (v) Segment of a circle is the region between the arc and chord of the circle.
- (vi) A circle divides the plane, on which it lies, in 3 (three) parts.

Q: 2: True/False -

- (i) Line segment joining the centre to any point on the circle is a radius of the circle.

Ans: True. Any line segment drawn from the centre of the circle to any point on it, is radius of circle and will be equal of length.

(ii) A circle has only finite of equal chords.  
Ans → False. There can be finite no. of equal chords of a circle.

(iii) If a circle is divided into three equal arcs, each is a major arc.

Ans → False. For unequal arc, there can be major or minor arc. So equal arc cannot be said as major or minor.

(iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.

Ans → True. Any chord whose length is twice as long as the radius of the circle, always passes through centre.

(v) Sector is a region between the chord and its corresponding arc.

Ans → False. A sector is a region of circle between the arc and the 2 radii of circle.

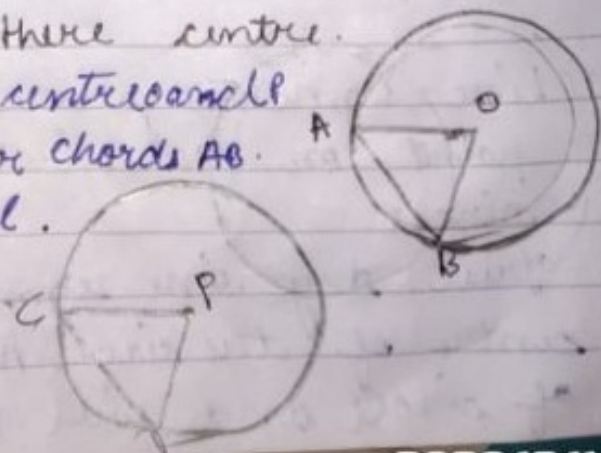
(vi) A circle is a plane figure.

Ans → True. A circle is a 2D fig and it can be drawn on plane surface.

### Ex → 10.2.

Q: 1. Recall that two circles are congruent, if they have the same radii, prove that equal chords of congruent circle subtend equal angles at their centre.

Ans → Given → Two circles with centres  $O$  and  $P$  are congruent. Their chords  $AB$  and  $CD$  are equal.



To prove:  $\angle AOB = \angle CPD$ .

Solution: In  $\triangle AOB$  and  $\triangle CPD$ .

$AO = CP$  [circles are congruent]  $\therefore$  (1)  
 $OB = PD$  [Therefore radii are equal] (1)

$AD = CD$  [Given] (2)

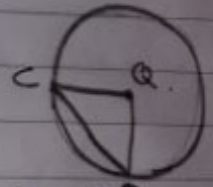
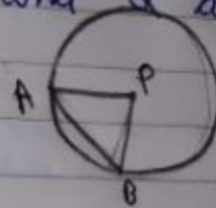
$\therefore \triangle AOB \cong \triangle CPD$  [S.S.S]

$\therefore \angle AOB = \angle CPD$

Q: 2: Prove that, if chords of congruent circle subtend equal angles at their centre, then the chords are equal.

Given: Two circles with centre P and Q are congruent  
 $\angle APB = \angle CQD$ .

To prove:  $AB = CD$



Solution In  $\triangle APB$  and  $\triangle CQD$ .

$AP = CQ$  [Radii of congruent circles] (1)

$\angle APB = \angle CQD$  [Given] (2)

$PB = QD$  [Radii of congruent circle] (3)

$\therefore \triangle APB \cong \triangle CQD$  [S.A.S]

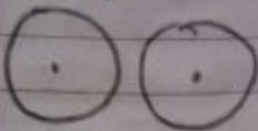
$\therefore AB = CD$

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Exercise = 10.3

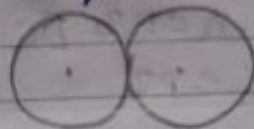
Q:1 Draw different pair of circles. How many points does each pair have in common? What is the maximum no. of common points?

Solution: I pair



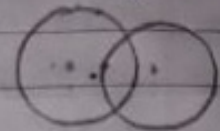
0 common points

II pair



1 common point

III pair

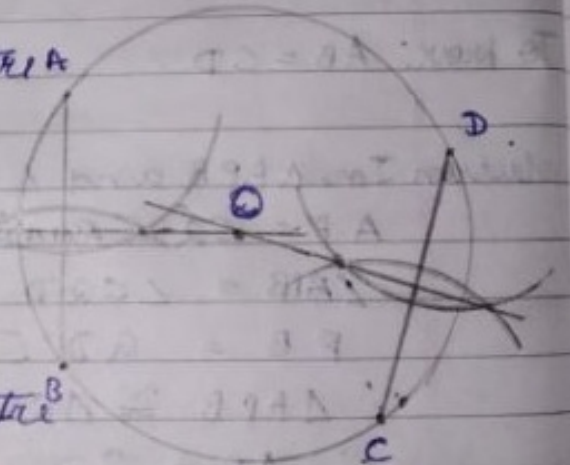


2 common points

Q:2 Construction suppose you are given a circle. Give a construction to find its centre.

Construction to find centre of a circle

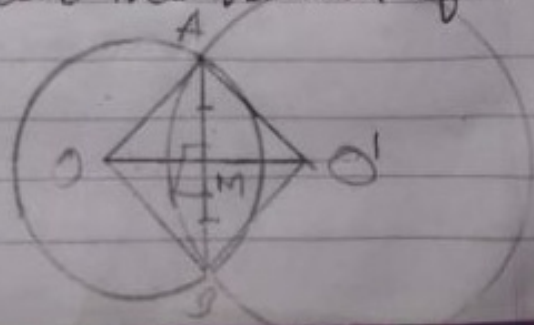
- 1 Draw two equal chords
- 2 Draw their perpendicular bisector
- 3 The point on which they intersect will be the centre of the circle.



Q:3 If two circles intersect at two points, prove that their centre lie on the bisector of common chord.

Given: AB is a common chord of 2 circles

Prove:  $\angle AMO = \angle BMO = 90^\circ$   
 $AM = BM$



Construction: Join OA and OB. [Joining the radius of both circle]  
Join O'A and O'B.

Proof: In  $\triangle AOO'$  and  $\triangle BOO'$   
 $OA = OB$  [radius of 1<sup>st</sup> circle]  
 $O'A = O'B$  [radius of 2<sup>nd</sup> circle]  
 $OO' = OO'$  [common]  
 $\triangle AOO' \cong \triangle BOO'$  [SSS]  
 $\angle AOO' = \angle BOO'$  [By CPCT]

In  $\triangle AMO$  and  $\triangle BMO$ .  
 $OA = OB$  [radius of circle]  
 $OM = OM$  [common]  
 $\angle AOM = \angle BOM$  [ $\angle AOO' = \angle BOO'$ ]  
 $\triangle AMO \cong \triangle BMO$  [SAS]  
 $AM = BM$  [CPCT]  
 $\angle AMO = \angle BMO$  [CPCT]  
 $\angle AMO + \angle BMO = 180^\circ$  [AB is line]  
 $\angle BMO + \angle BMO = 180^\circ \rightarrow 2\angle$   
 $= 2\angle BMO = 180^\circ$   
 $\angle BMO = \angle AMO$   
 $\angle BMO = \angle AMO = 90^\circ$

Exercise = 10.4.

Q.2 Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

Solution In  $\triangle OAO'$

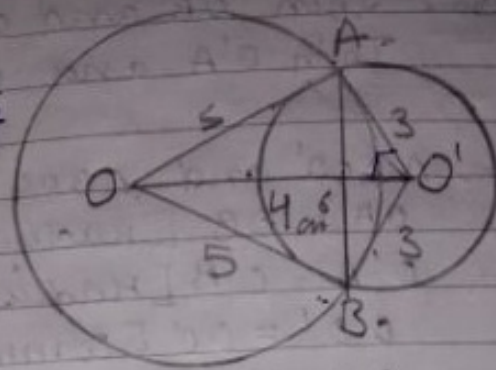
$$OA^2 = (5)^2 = 25 \text{ cm}$$

$$(OO')^2 + (AO')^2 = (4)^2 + (3)^2$$

$$= 16 + 9 = 25$$

By Pythagorean theorem

$$\angle AOO' = 90^\circ \text{ --- (i)}$$



In  $\triangle OBO'$

$$OB^2 = (5)^2 = 25 \text{ cm}$$

$$(OO')^2 + (BO')^2 = (4)^2 + (3)^2$$

$$= 16 + 9 = 25 \text{ cm}$$

$$OB^2 = (OO')^2 + (BO')^2$$

$$\angle BO'O = 90^\circ \text{ [Pythagorean theorem] --- (ii)}$$

Adding eq. (i) and (ii)

$$\angle AO'O + \angle BO'O = 180^\circ$$

$$90 + 90 = 180$$

$$\angle AO'B = 180^\circ$$

$AO'B$  will be a straight line.

$$AB = OA + O'B$$

$$= 3 + 3 = 6 \text{ cm}$$

$$\boxed{AB = 6 \text{ cm}}$$

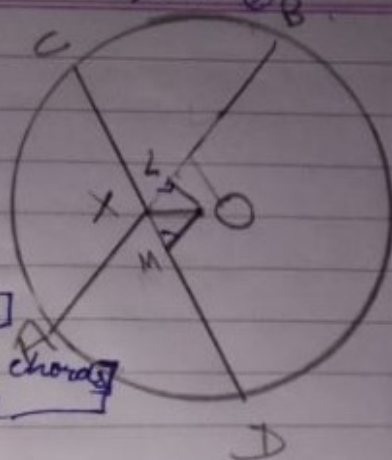
Q.2: If two chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Given:  $AB = CD$ , and intersect at  $X$ .

To prove:  $AX = DX$ ,  $CX = AX$ .

Construction: Draw  $OL \perp AB$   
 $OM \perp CD$ .

Proof: In  $\triangle O LX$  and  $\triangle O M X$ .  
 $\angle O L X = \angle O M X [90^\circ]$ .  
 $OX = OX$  [Common]  
 $BL = MD$  [ $\perp$  on equal chords are equal].  
 $\triangle O L X \cong \triangle O M X$  [RHS].  
 $LX = MX$  [CPCT]. - ①



$AB = CD$  [Given].

$$\frac{1}{2} AB = \frac{1}{2} CD$$

$BL = MD$  [Perpendicular from centre bisects the chord] ②

Adding eq. ① and ②.

$$\begin{array}{r} [LX = MX] \\ + [BL = MD] \\ \hline [BX = DX] \end{array}$$

$$AB = CD$$

$$- BX = DX$$

$$\boxed{AX = CX}$$

$$AX = CX$$

1.3. If two equal chords of a circle intersect within the circle, prove that the joining the points of intersection to the centre makes point equal angles with chords.



Given:  $AB = CD$  chords intersect at  $X$ .

To prove:  $\angle OXL = \angle OXM$

Construction:  $OL \perp AB$ ,  $OM \perp CD$ .

Proof  $\rightarrow$  In  $\triangle O LX$  and  $\triangle O M X$ .

$$\angle O LX = \angle O M X \quad [90^\circ] \quad R$$

$$OX = OX \quad [\text{Common}] \quad H$$

$OL = OM$  [equal chords are equidistance from centre]  $S$ .

$$\triangle O LX \cong \triangle O M X \quad [RHS]$$

$$\angle OXL = \angle OXM \quad [C.P.C.T.]$$

Q.4: If a line intersects two concentric circles with centre  $O$  at  $A, B, C, D$ , prove  $AB = CD$ .

Proof  $\rightarrow OM \perp AD$ .

Then  $\rightarrow BM = CM$  [Perpendicular from centre bisects the chord]  $\text{---} \textcircled{1}$

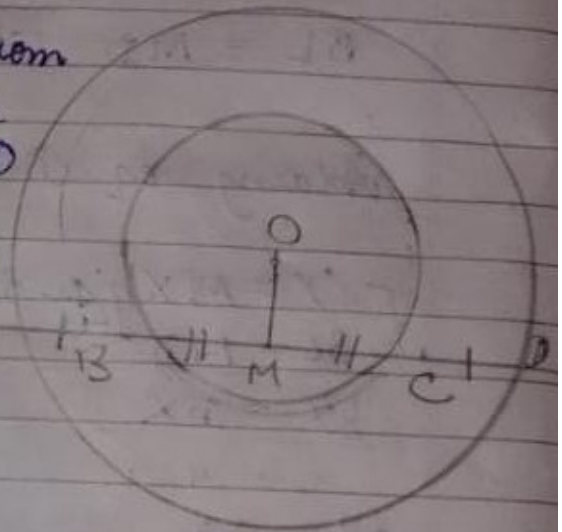
$$AM = DM - \textcircled{1} \quad [10.3 \text{ Thm.}]$$

subtracting eq.  $\textcircled{1}$  from  $\textcircled{2}$ .

$$AM = DM$$

$$- BM = CM$$

$$\underline{AB = CD.}$$



5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and

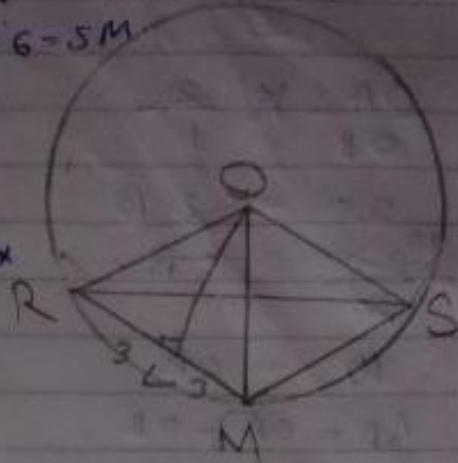
Salma and between Salma and Mandip is 6m each. What is distance between Rishma and Mandip?

Given =  $OR = OM = OS = 5m$ ,  $RS = 6 = 5M$

Find =  $RM$ .

Let  $\rightarrow OL \perp RS$ .

Let  $\rightarrow R = 6$ ,  $RL = 3$  [ + from centre bisector the chord



In  $\triangle OLR$

$$OR^2 = RL^2 + OL^2$$

$$5^2 = 3^2 + OL^2$$

$$25 = 9 + OL^2$$

$$25 - 9 = OL^2$$

$$\sqrt{16} = \sqrt{OL^2} = 4$$

$$OL = 4$$

$$\text{ar. of } ORS = \frac{1}{2} \times B \times P.$$

$$= \frac{1}{2} \times 6 \times 4 = \frac{1}{2} \times 5 \times RN$$

$$\frac{24}{5} = RN = 4.8 \text{ m.} = \textcircled{a}$$

$$RN = MN$$

$$RM = \cancel{8} \text{ m } RN + MN$$

$$RM = 4.8 + 4.8 = 9.6 \text{ m}$$

Q: 6. A circular park of radius 20m is situated in a colony. Three boys Ankur, Ayad and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the strings of each phone.

Solution:

$$AS = SD.$$

$$AS = 2BS.$$

$$\frac{AS}{2} = BS. \quad \text{--- (1)}$$

$$\frac{OA}{OB} = \frac{2}{1}$$

$$20 = 2OB$$

$$\frac{10}{2} = OB.$$

$$10 = OB.$$

$$AB = OA + OB.$$

$$= 20 + 10 = 30.$$

$$AB = 30.$$

By Pythag. Theorem.

$\Delta ABS$

$$AS^2 = AB^2 + BS^2.$$

$$= (30)^2 + \left(\frac{AS}{2}\right)^2$$

$$AS^2 = 900 + \frac{AS^2}{4}$$

$$AS^2 = \frac{3600 + AS^2}{4}$$

$$4AS^2 = 3600 + AS^2.$$

$$4AS^2 - AS^2 = 3600.$$

$$3AS^2 = 3600.$$

$$AS^2 = \frac{3600}{3} = 1200.$$

$$\sqrt{AS^2} = \sqrt{1200} = 20\sqrt{3}.$$

$$\text{Ans} = 20\sqrt{3} \text{ m}.$$

