Ch = 9

Area of Parallelogram and Tri...

Ex. → 9.1

Q:1. Which of the following fig. lie on the same base and between the same parallels.

(i) Trapezium ABCD and ~~APDC~~ ΔPDC lies on the same in between the same parallel lines AB and DC.

(ii) Parallelogram PQRS and trapezium SMNR lie on the same base SR but not in between the same parallel lines.

(iii) Parallelogram PQRS and ΔRTQ lies on the same base QR and in between the same parallel lines QR and PS.

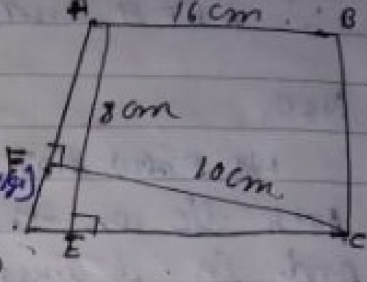
(iv) Parallelogram ABCD and ΔPQR do not lies on the same base but in between the same parallel lines BC and AD.

(v) Quadrilateral ABQD and trapezium APCD lie on the same base ~~and~~ in between the same parallel lines AB and BQ.

(vi) Parallelogram PQRS and parallelogram ABCD do not lie on the same base and.

Ex. → 9.2

Q.1. In the given fig, ABCD is parallogram, $AE \perp DC$ and $CF \perp AD$. AB = 16 cm, AE = 8 cm and CF = 10cm. find AD.



Ans → Given,

AB = CD = 16 cm (opp. sides of llgm)

CF = 10cm, AE = 8 cm.

Now,

Area of Parallelogram = Base × P.H.

$CD \times AE = AD \times CF$

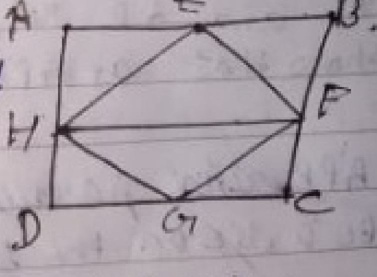
$16 \times 8 = AD \times 10cm$

$128 = AD \times 10$

$AD = 128/10$

$AD = 12.8$ cm

Q.2. If E, F, G and H are respectively the mid points of the sides of parallelogram ABCD, show that they are (EFGH) = 1/2 ar (ABCD).



Ans. Given,

EFG and H are the mid points of the sides of a parallelogram ABCD respectively.

To Prove –

ar (EFGH) = 1/2 ar (ABCD).

Construction H and F are joined.

Proof.

AD ∥ BC and AD = BC (opp. side of llgm)

⇒ $\frac{1}{2}AD = \frac{1}{2}BC$.

Also,

AH || BF and DH || CF

$\Rightarrow$ AH = BF and DH = CF. (H and F are mid points)

$\therefore$ ABFH and HFCD are parallelogram.

Now,

We know that, Δ EFH and parallelogram AB both lie on the same FH the common base and in between the same parallel lines AB and HF

$\therefore$ Area of EFH = 1/2 Area of ABFH — (i).

And, area of GHF = 1/2 Area of HFCD — (ii).

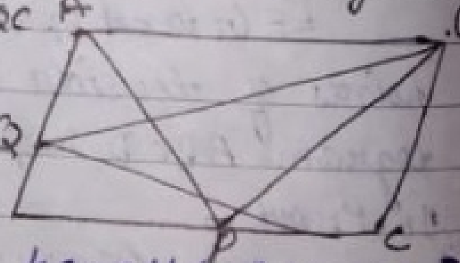
Adding (i) and (ii).

Area of Δ EFH + Area of Δ GHF = 1/2 Area of ABFH + 1/2 Area of HFCD

= Area of EFGH = Area of ABFH.

$\therefore$ Ar(EFGH) = $\frac{1}{2}$ Ar of (ABCD.) Hence, proved

Q:3. P and Q are any two points lying on the side DC and AD respectively of a parallelogram ABCD show that ar APB = Ar BQC



Ans. Δ APB and parallelogram ABCD lie on the same Base and in between same parallel AB and DC.

Ar (ΔAPB) = 1/2 Area (parallelogram ABCD — (i)

Similarly

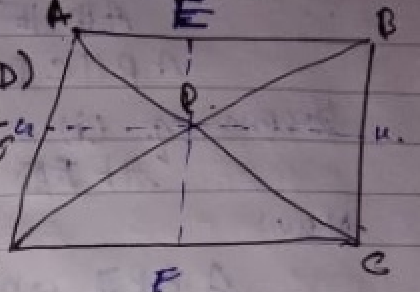
Ar (ΔBQC) = 1/2 (Parallelogram ABCD — (ii)

From (i) and (ii)

Ar ($\triangle APB$) = Ar ($\triangle BQC$)

H.P.

Q.4. If fig. 9.16, is a point in the interior of a parallelogram ABCD, show that.



(i) Ar (APB) + ar (PCD) = ar (ABCD)

ar (APD) + ar (PBC) = ar (APB) + ar (PCD)

Hint: through P, draw a line // to AB

Ans (i) A line GH is drawn parallel to AB passing through P

In a parallelogram,

AB ∥ GH (by construction) ---- (i)

∴

AD ∥ BC = AG ∥ BH ---- (ii)

From eq. (i) and (ii)

ABGH is a parallelogram.

Now,

$\triangle APB$ and parallelogram ABHG are lying on the same base AB and in between the same parallel lines AB and GH.

∴ Ar ($\triangle APB$) = 1/2 ar (ABHG) ---- (iii).

Also,

$\triangle PCD$ and parallelogram CDGH are lying on the same base CD and in between the same parallel lines CD and GH.

∴ ar ($\triangle PCD$) = 1/2 ar (CDGH) ---- iv.

ar ($\triangle APB$) + ar ($\triangle PCD$) = 1/2 [ar (ABHG) + ar (CDGH)]

= ar (APB) + ar (PCD) = 1/2 ar (ABCD)

(ii) ~~A~~ line EF is drawn parallel to AD passing through P.

In the parallelogram.

AD ∥ EF (by construction) --- (i)

∴, ~~AD ∥ EF~~

AB ∥ CD ⇒ AE ∥ DF --- (ii)

From eq. (i) and (ii)

AE~~FD~~ is a parallelogram

Now.

△APD and parallelogram AEFD are lying on the same base AD and in between the same parallel lines AD and EF.

∴ ar(△APD) = 1/2 (AEFD) ------ (iii)

Also,

△PBC and parallelogram BCFE are lying on the same base BC and in between the same parallel lines BC and EF.

∵ ar (△PBC) = 1/2 ar (BCFE) ....(iv)

Adding eq. (iii) and (iv)

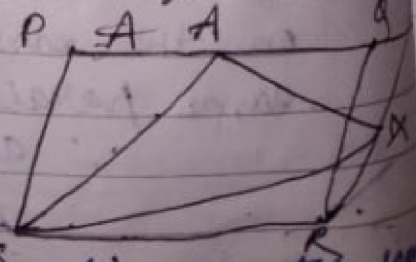
ar (△APD) + ar (△PBC) = 1/2 [ar(AEFD) + ar (BCFE)]

⇒ ar (APD) + ar (PBC) = ar (APB) + ar (PCD)

Q:5. In the given fig. PQRS and ABRS are parallelograms and X is any point on side BR. show

(i) ar PQRS = ar ABRS.

(ii) ar AXS = $\frac{1}{2}$ ar PQRS.



Ans(i) Parallelogram PQRS and ABRS lie on the same base SR in-between the same parallel lines

SR and PB.

$\therefore$ ar (PQSR) = ar (ABS) ---- (i)

(ii) ΔAXS and parallelogram ABRS are lying on same base AS and b/w the same parallel lines AS and BR.

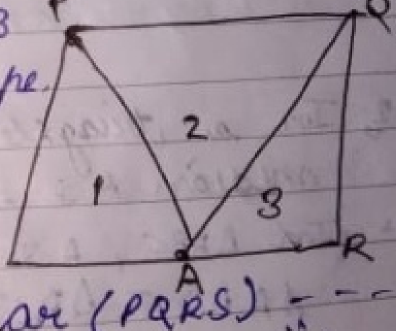
$\therefore$ ar (Δ AXS) = 1/2 ar (ABRS) ---- (ii).

From eq. (i) and (ii), we find that,

ar(Δ AXC) = 1/2 ar(PQRS).

Q.6. A famer was having a field in the form of parallelogram PQRS she took point A on RS and joined it to points P and Q. In how many parts the field is divided? what are the shaps of these parts? the farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it

Ans → The field is divided into 3 parts each in triangle shape.



Let, ΔPSA, ΔPAQ and ΔQAR be the triangle.

Ar of ΔPSA + ΔPAQ + ΔQAR = ar (PQRS) ---- (i)

Δ PAQ = 1/2 Area of PQRS. ---- (ii).

Here the Δ and ||gram are on the same base and in-between the same parallel lines.

From eq. (i) and (ii).

Area of ΔPSA + Area of Δ QAR = 1/2 of Area of PQRS. ---- (iii).

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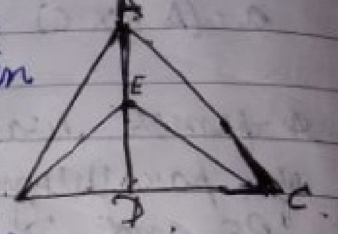
From (ii) and (iii), we can conclude that
The farmer ~~most~~ must sow wheat or pulses in ΔPAQ or either in both ΔPSA and ΔQAR.

Ex. 9.3

Q.1. In fig. E is any point on median AD of a ΔABC. Show that ar(ADE) = ar(ACE).

Ans. In ΔABC, AD is a median and it will divide ΔABC in two equal parts.



$\therefore \Delta ABD = \Delta ACD$ —— (1)

In ΔEBC, ED is the median it will divide ΔEBC in 2 equal parts.

$\therefore \Delta EBD = \Delta EDC$ —— (2)

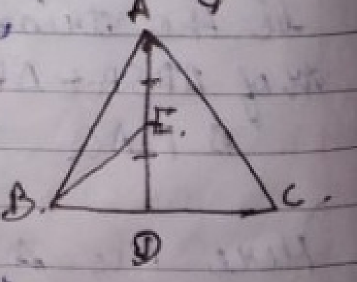
Subtract (2) eq. from (1)

$\Delta ABE = \Delta ACE$

Hence, proved.

Q.2. In a triangle ABC, E is the mid-point of median AD. Show that ar(BED) $= \frac{1}{4}$ ar(ABC)

Ans. In ΔABC, AD is the median,



$\Delta ABD = \Delta ACD$

$\Delta ABC = \Delta ABD + \Delta ACD$

$= \Delta ABD + ABD$

$\Delta ABC = 2\,\Delta ABD$

$\frac{1}{2} \Delta ABC = \Delta ABD$ —— (i)

In ΔABD, BE is the median [AE = DE].

$\therefore \Delta BEA = \Delta BED$.

vivo Y20 $\Delta ABD = \Delta BEA + \Delta BED$. 2020.12.07 14:55

$= \Delta BED + \Delta BED$.

$\Delta ABD = 2\ BED$.

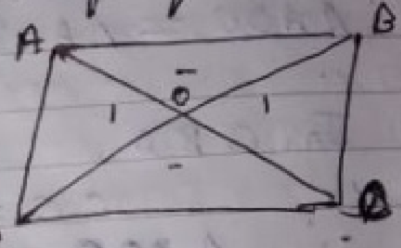
$\frac{1}{2} ABD = \Delta BED$.

$\frac{1}{2} \times \frac{1}{2}\ \Delta ABC = \Delta BED$.

$\frac{1}{4} ABC = \Delta BED$..

$\frac{1}{4}$ ar ABC = ar BED (H.P.).

Q:3. Show that diagonals of a parallelogram divide it into 4 triangle of eq. area.



Ans → Given,

A ∥ gram ABCD with diagonals AC and BD.

To prove,

$AOB = BOC = COD = DOA$.

Proff → The diagonals of parallelogram are bisect each other.

In ΔABC,

$AO = AC$, the OB is the median

$\Delta AOB = \Delta BOC$ —— ①.

In ΔBCD,

$OB = OD$, OC is the median

$\Delta BOC = \Delta COD$ — ②

In ΔACD,

$OA = OC$.., OD is the median.

$\Delta AOD = \Delta COD$ —— ③.

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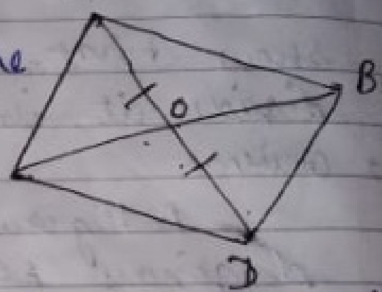
By ①, ② and ③ eq., we conclude that,
$\Delta AOB = \Delta BOC = \Delta COD = \Delta DOA$.
The diagonal of a parallelogram [H.P.]
divide its 4 triangle in equal part.

Q:4. In the given fig. ABC and ABD are 2 triangles on the same AB. If line segment CD is bisected by AB at O. show that ar(ABC) = ar(ABD). [H.P.]

Ans: In ΔADC,



OC = OD, then AO is the median.
$\therefore \Delta AOC = \Delta AOD$. —— ①.

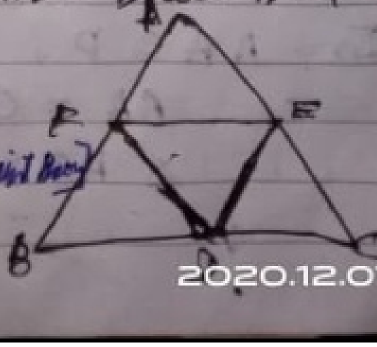
In CBD,
OC = OD, then OB is the median.
$\therefore \Delta BOC = \Delta BOD$. —— ②.

Adding ① and ② eq.–

$\Delta AOC + \Delta BOC = \Delta ABC$
$\Delta AOD + BOD = \Delta ABD$.
$\therefore \Delta ABC = \Delta ABD$.
$ar(\Delta ABC) = ar(\Delta ABD)$.
[H.P.]

Q.5. D, E and F are respectively the mid-points of BC, CA and AB of a ΔABC show that

(i) BDEF is a parallelogram.



i) Ans: $FE \parallel BC$, $FE = \frac{1}{2} BC$. [Mid point theorem]
$BD = \frac{1}{2} BC$. [MPT]

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FE ∥ BD, FE = BD. [opp. sides are equal and ∥]
∴ BDEF is ∥gram. —— (i).

(ii) ar (DEF) = $\frac{1}{4}$ ar (ABC).

similarly are qu. (i), also.
DCEF, DEAF, BDEF — A parallelogram.

In parallelogram DCEF,
DE line segment divide it into 2 equal part.
That is →
$\Delta DEF = \Delta DEC$ —(i)

In parallelogram DEAF,
FE is divide it into 2 equal parts
∴ $\Delta DEF = \Delta AEF$ —(ii)

In BDEF, FD is divide it into 2 parts.
∴ ΔDEF = BDF. —— (iii)

ar(Δ ABC) = ar(Δ AEF) + ar(Δ BDF) + ar(Δ DEF) + ar(Δ DCE)

By eq. (i), (ii) and (iii), we ~~find to~~ conclude that

Δ ABC = DEF + DEF + DEF + DEF
Δ ABC = 4 DEF.
$\frac{1}{4}$ ar(ΔABC) = ar(ΔDEF).

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(iii) ar(BDEF) $= \frac{1}{2}$ ar(ABC).

Ans → BDEF = ΔDEC + ΔBDF

BDEF = ΔDEF + ΔDEF.

= 2ΔDEF.

BDEF = $2 \times \frac{1}{4}$ ar of ΔABC.

BDEF = $\frac{1}{2}$ ar of ABC.

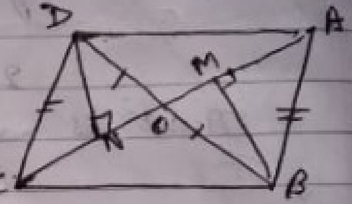
Q:6 ABCD is a quadrilateral. Diagonals AC and BD intersect at O. OB = OD, AB = CD.

Show that:

(i) ar(DOC) = ar(AOB).

(ii) ar(DCB) = ar(ACB)

(iii) DA∥CB or ABCD is ∥gram.



Const: Draw DN ⊥ AC and BM ⊥ AC.

Ans(i) In ΔDON and ΔBOM

$\angle$DON = $\angle$BOM [V.O.A.] A.

$\angle$N = $\angle$M [each 90°] A.

DO = BO [given] S.

∴ ΔDON ≅ ΔBOM [A.A.S.]

⇒ ar(DON) = ar(BOM) —— ①.

In ΔDCN and ΔBAM.

$\angle$N = $\angle$M [each 90°] R

CD = AB [given] H

DN = BM [∵ ΔDON ≅ ΔBOM, DN = BM (by cpct)] S.

∴ ΔDCN ≅ ΔBAM (R.H.S.)

ar(DCN) = ar(BAM) —— ②.

Adding ① and ②.

ar(DON) + ar(DCN) = ar(BOM) + ar(BAM).

ar(DOC) = ar(AOB)

H.P.

Ans(ii) From (i) question =

$ar(DOC) = ar(AOB)$.

adding $ar(BOC)$ on both side

$ar(DOC) + ar(BOC) = ar(AOB) + ar(BOC)$.

$ar(DCB) = ar(ACB)$.

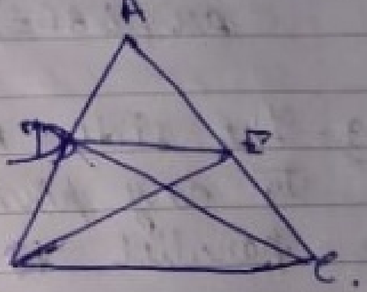
Ans(iii) ΔDCB and ΔACB lie on the same base BC and ~~in between~~ are equal in area.

∵ They lie between same parallels.

→ $AD \parallel BC$.

∴ ABCD is a ∥gram.

Q.7 D and E are points on side AB and AC respectively of ΔABC such that $ar(DBC) = ar(EBC)$. Prove that $DE \parallel BC$.

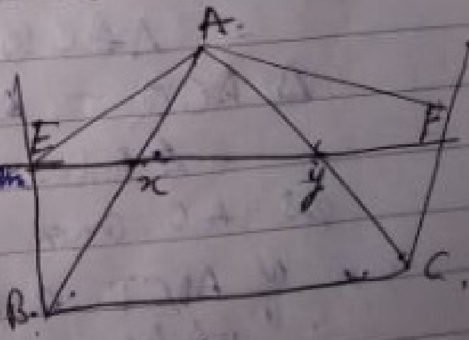


Ans → ΔDBC and ΔEBC are on same base BC, and equal in area.

then, the triangles lie between parallel lines.

∴ $DE \parallel BC$

Q.8 XY is a line parallel to side BC of triangle ABC. If $BE \parallel AC$ and $CF \parallel AB$ meet XY at E and F respectively. show that $ar(ABE) = ar(ACF)$



Ans → $xy \parallel BC$ [given] → $EY \parallel BC$.

$BE \parallel AC$ [,,] ⇒ $BE \parallel CY$

Therefore BCYE is a parallelogram

$xy \parallel BC$ → $xF \parallel BC$.

$CF \parallel AB$ → $BX \parallel CF$

Therefore BCFX is also a parallelogram.

BCYE and BCFX are on same base and between same parallels EF and BC.

$\therefore$ ar(BCYE) = ar(BCFX). —— ①

ΔABE and BCYE are on same base BE and b/w same parallels BE and AC.

$\therefore$ $\Delta ABE = \frac{1}{2} BCYE$. —— ②

ΔACF and BCFX are on same base CF and b/w same parallel lines AD and CF

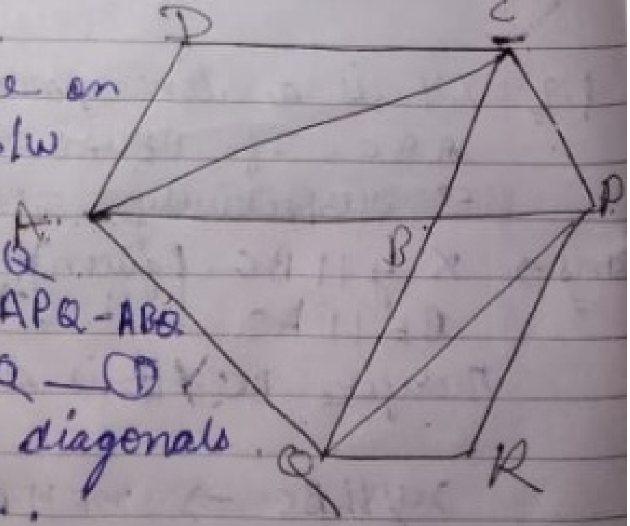
$\therefore$ $\Delta ACF = \frac{1}{2} BCFX$. —— ③

By eq. ①, ②, ③ we conclude that.

ar(ΔABE) = ar(ΔACF).

Q:9 - The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed. Show that ar (ABCD) = ar(PBQR).

Sol.→ Join AC and PQ.



ΔAPQ and ΔACQ are on same base AQ and b/w same ||s PQ and CP.

$\therefore$ $\Delta ACQ = \Delta APQ$

$\Delta ACQ - \Delta ABQ = \Delta APQ - ABQ$

$\Delta ABC = \Delta PBQ$ —— ①

As AC and PQ are diagonals of ABCD and PBQR.

$\Delta ABC = \frac{1}{2} ABCD$

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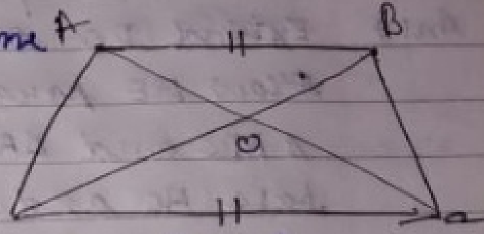
$\Delta PBQ = \frac{1}{2} PBQR$ — ②.

$\Delta ABC = \Delta PBQ$.

$\frac{1}{2} ABCD = \frac{1}{2} PBQR$.

$ar(ABCD) = ar(PBQR)$

Q.10: Diagonals AC and BD of a trapezium ABCD with AB ∥ CD intersect each other at O. Prove that $ar(AOD) = ar(BOC)$.



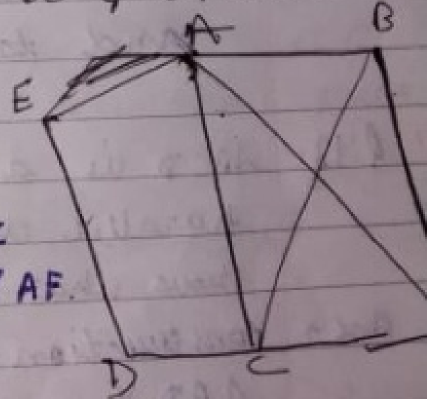
Ans: ΔDAC and ΔCBD are on same base DC and in b/w same parallels DC and AB.

$\therefore ar(\Delta DAC) = ar(\Delta CBD)$

$ar(\Delta DAC) - ar(\Delta DOC) = ar(\Delta CBD - ar(\Delta DOC))$

$ar(\Delta AOD) = ar(\Delta BOC)$.

Q.11 In the giv fig ABCDE is a pentagon, A line through B parallel to AC meets DC produced at F. Show that



(i) $ar(ACB) = ar(ACF)$

(ii) $ar(AEDF) = ar(ABCDE)$

(i) ΔACB and ΔACF are on same base ~~AC~~ BC $\boxed{\text{given } AC \parallel BF}$ and b/w same ∥s AC & AF.

$\therefore ar(ACB) = ar(ACF)$ — (i)

(ii) By eq. (i).

$ar(ACB) = ar(ACF)$

$ar(ACB) + ar(ACDE) = ar(ACF) + ar(ACDE)$

$ar(ABCDE) = ar(AEDCF)$

$ar(ABCDE) = ar(AEDF)$.

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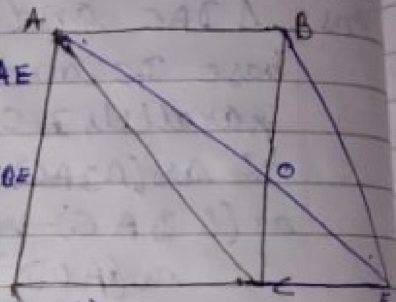
Q:12. A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

Ans → Extend DC to point E.

Draw BE parallel to AC, Join AE



Δ ABC and ΔAEC are on same base AC and b/w same ||s AC and BE

$\therefore$ ar(ΔABC) = ar(AEC).

ar(ABC) − ar(ΔAOC) = ar(ΔAEC) − ar(AOC)

ar(ΔAOB) = ar(ΔCOE).

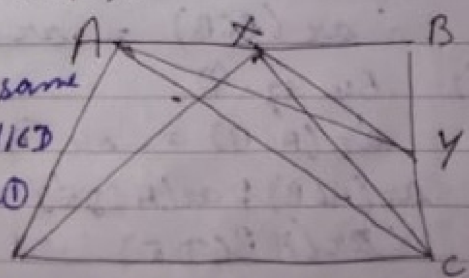
Conclusion −

ΔAOB → Given by Itwaari to you.

and take COE to make Triangular plot.

Q:13. ABCD is a trapezium with AB || DC. A line parallel to AC intersect AB at X and BC at Y. Prove that ar(ADX) = ar(ACY).

Ans → construction − Join XC.



ΔADX and ΔACX are on same base AX and b/w same ||s AB || CD

$\therefore$ ar(ΔADX) = ar(ΔACX) −①

ΔACY and ΔACX are on same base AC and in b/w same ||s → AC || XY.

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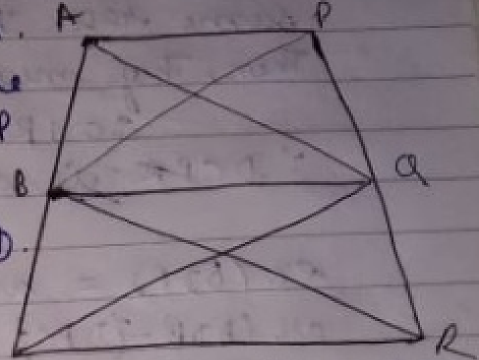
∵ ar(△ACY) = ar(△ACX) — ③.
By ① and ③ :
∵ ar(ADX) = ar(ACY).

Q.14. In the giv fig AP||BQ||CR. Prove that ar(AQC) = ar(PBR)

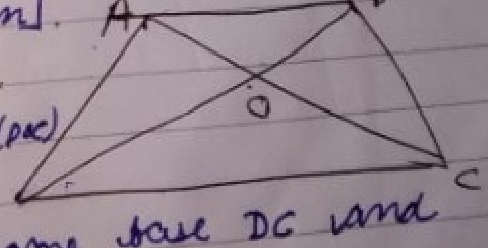
Ans → In Quadrilateral APQB, AP||BQ.



△ABQ and △PBQ are on same base BQ and b/w same ||s AP and BQ.
∵ ar(△ABQ) = ar(△PBQ) —①.
In qua. BQRC, BQ||CR.
△BQC and △BQR are on same Base BQ and in b/w same ||s - BQ and CR
∴ ar(△BQC) = ar(BQR) — ②.
Add eq. ① and ②.
ar(△ABQ) = ar(△PBQ) — ①.
+ ar(△BQC) = ar(△BQR) — ②
ar(△AQC) = ar(△PBR), Hence, proved.

Q.15. Diagonals AC and BD of a qua ABCD intersect at O in such a way that ar(AOD) = ar(BOC). Prove that ABCD is a trapezium.

Ans → ar(AOD) = ar(BOC) [given].



Adding Both sides by △DOC.
ar(AOD) + ar(DOC) = ar(BOC) + ar(DOC)
ar(ACD) = ar(BCD)
△ACD and △BCD are on same base DC and ~~b/w same~~ ||s equal in area
∵ Both lie b/w parallel lines.
∴ AB||CD.
∴ ABCD is a trapezium.

Q:16- In the giv fig, ar(DRC) = ar(DPC) and ar(BDP) = ar(ARC). show that both the qua. ABCD and DCPR are trapeziums.

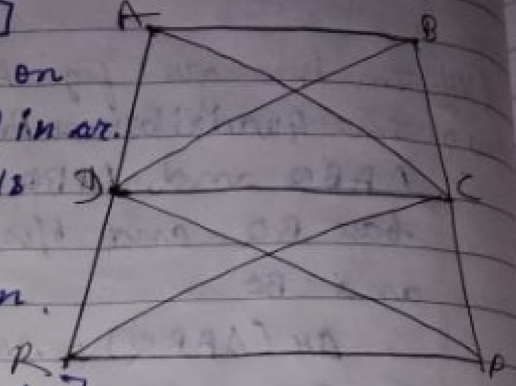
Ans→ ar(DRC) = ar(DPC) [giv]

ΔDRC and ΔDPC are on same base. DC and equal in ar.

Then, They must lie b/w ||s

DC || RP

∴ DCPR is a trapezium.



ar(BDP) = ar(ARC). [giv]

ar(BDP) − {DPC} = ar(ARC) − DRC.

ar(BDC) = ar(ADC).

ΔBDC and ΔADC are on same base DC and equal in area.

Then, They must lie between ||s.

AB || DC

∴ ABCD is a trapezium